Collins

PHYSICS

AQA A-level Year 1 and AS
Student Book

Dave Kelly

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Approval Message from AQA

This textbook has been approved by AQA for use with our qualification. This means that we have checked that it broadly covers the specification and we are satisfied with the overall quality. Full details for our approval process can be found on our website.

We approve textbooks because we know how important it is for teachers and students to have the right resources to support their teaching and learning. However, the publisher is ultimately responsible for the editorial control and quality of this book.

Please note that when teaching the AS and A-level Physics course, you must refer to AQA's specification as your definitive source of information. While this book has been written to match the specification, it cannot provide complete coverage of every aspect of the course.

A wide range of other useful resources can be found on the relevant subject pages of our website: www.aqa.org.uk

CONTENTS

TO THE STUDENT

The aim of this book is to help make your study of advanced physics interesting and successful. It includes examples of modern applications, of new developments, and of how our scientific understanding has evolved.

Physics is our attempt to understand how the Universe works. Fortunately, there are some deep, underlying laws that simplify this ambitious task, but the concepts involved are often abstract and will be unfamiliar at first. Getting to grips with these ideas and applying them to solving problems can be daunting. There is no need to worry if you do not 'get it' straight away. Discuss ideas with other students, and of course check with your teacher or tutor. Most important of all, keep asking questions.

There are a number of features in the book to help you learn:

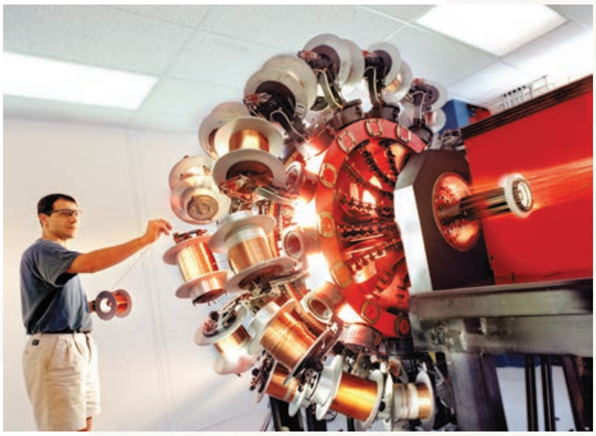
- Each chapter starts with a short outline of what you should have learned previously and what you will learn through the chapter. This is followed by a brief example of how the physics you will learn has been applied somewhere in the world.
- Important words and phrases are given in bold when used for the first time, with their meaning explained. There is also a glossary at the back of the book. If you are still uncertain, ask your teacher or tutor because it is important that you understand these words before proceeding.
- Throughout each chapter there are many questions, which enable you to quickly check your understanding. The answers are at the back of the book. If you get really stuck with a question, check the answer before you carry on.
- Similarly, throughout each chapter there are checklists of Key Ideas that summarise the main points you need to learn from what you have just read.
- Where appropriate, Worked examples are included to show how important calculations are done.
- There are many Assignments throughout the book. These tasks are designed to consolidate or extend your understanding of a topic. They give you a chance to apply the physics you have learned to new situations and to solve problems that require a mathematical approach. Some refer to practical work and will encourage you to think about scientific methods. The relevant Maths Skills (MS) and Practical Skills (PS) from the AQA AS Physics specification are indicated.
- Some chapters have information about the 'Required practical' activities that you need to carry out during your course. These sections (printed on a beige background) provide the necessary information about the apparatus, equipment and techniques that you need to carry out the required practical work. There are questions about the use of equipment, techniques, improving accuracy in practical work, and data analysis.

This book covers the requirements of AQA AS Physics and the first year of AQA A-level Physics. There are a number of sections, questions, Assignments and Practice Questions that have been labelled 'Stretch and challenge', which you should try to tackle if you are studying for A-level. In places these go beyond what is required for your exams but they will help you to expand your knowledge and understanding of physics.

Good luck and enjoy your studies. We hope this book will encourage you to study physics further after you have completed your course.

PRACTICAL WORK IN PHYSICS

Practical work is a vital part of physics. Physicists apply their practical skills in a wide variety of contexts: from nuclear medicine in hospitals to satellite design; from testing new materials to making astronomical observations. In your AS or A-level physics course you need to learn, practise and demonstrate that you have acquired these skills.



Physicists need to solve problems, such as design problems. This machine weaves superconducting wire into cable to produce powerful superconducting electromagnets for accelerators.

WRITTEN EXAMINATIONS

Your practical skills will be assessed in the written examinations at the end of the course. Questions on practical skills will account for about 15% of your marks at AS and 15% of your marks at A-level. The practical skills that will be assessed in the written examinations are listed below. Throughout this book there are questions and longer assignments that will give you the opportunity to develop and practise these skills. The contexts of some of the exam questions will be based on the 'required practical activities' (see the final section of this chapter).

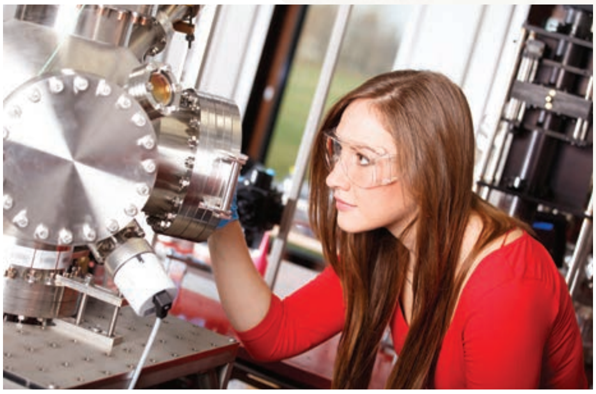
Practical skills assessed in written examinations:

Independent thinking

- Solve problems set in practical contexts
- Apply scientific knowledge to practical contexts

Use and application of scientific methods and practices

- Comment on experimental design and evaluate scientific methods

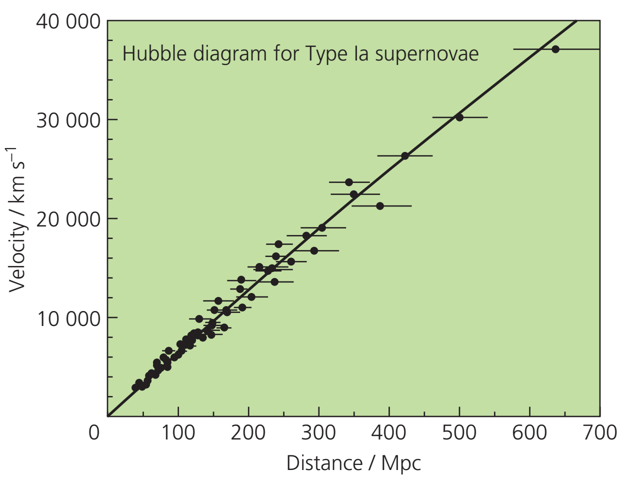


Physicists need to apply their knowledge when using practical equipment. This is a laser deposition chamber, in which a laser beam evaporates material in order to coat another surface.

- Present data in appropriate ways
- Evaluate results and draw conclusions with reference to measurement uncertainties and errors
- Identify variables, including those that must be controlled

Numeracy and the application of mathematical concepts in a practical context

- Plot and interpret graphs
- Process and analyse data using appropriate mathematical skills
- Consider margins of error, accuracy and precision of data



This graph of velocity against distance for supernova events, similar to that originally produced by Edwin Hubble, plots the distances with error bars because of the uncertainty in the values.

Instruments and equipment

- Know and understand how to use a wide range of experimental and practical instruments, equipment and techniques appropriate to the knowledge and understanding included in the specification

You will need to use a variety of equipment correctly and safely.

ASSESSMENT OF PRACTICAL SKILLS

Some practical skills, such as handling materials and equipment and making measurements, can only be practised when you are doing experiments. For A-level, the following *practical competencies* will be assessed by your teacher when you carry out practical activities:

- Follow written procedures
- Apply investigative approaches and methods when using instruments and equipment
- Safely use a range of practical equipment and materials
- Make and record observations
- Research, reference and report findings

You must show your teacher that you consistently and routinely demonstrate the competencies listed above during your course. The assessment will not contribute to your A-level grade, but will appear as a 'pass' alongside your grade on the A-level certificate.

These practical competencies must be demonstrated by using a specific range of *apparatus and techniques*. These are as follows:

- Use appropriate analogue apparatus to record a range of measurements (to include length/distance, temperature, pressure, force, angles and volume) and to interpolate between scale markings
- Use appropriate digital instruments, including electrical multimeters, to obtain a range of measurements (to include time, current, voltage, resistance and mass)
- Use methods to increase accuracy of measurements, such as timing over multiple oscillations, or use of a fiduciary marker, set square or plumb-line
- Use a stopwatch or light gates for timing
- Use calipers and micrometers for small distances, using digital or vernier scales
- Correctly construct circuits from circuit diagrams using dc power supplies, cells and a range of circuit components, including those where polarity is important
- Design, construct and check circuits using dc power supplies, cells and a range of circuit components
- Use signal generator and oscilloscope, including volts/division and time-base

- Generate and measure waves, using microphone and loudspeaker, or ripple tank, or vibration transducer, or microwave/radio wave source
- Use laser or light source to investigate characteristics of light, including interference and diffraction
- Use ICT such as computer modelling or data logger with a variety of sensors to collect data, or use software to process data
- Use ionising radiation, including detectors

For AS, the practical competencies will not be assessed, but you will be expected to use these skills and these types of apparatus to develop your manipulative skills and your understanding of the processes of scientific investigation.

REQUIRED PRACTICAL ACTIVITIES

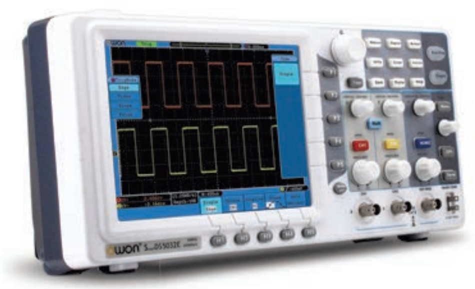
During the A-level course you will need to carry out 12 *required practical activities*. These are the main sources of evidence that your teacher will use to award you a 'pass' for your competency skills. If you are studying the AS course, you will need to carry out the first six in this list.

1. Investigation into the variation of the frequency of stationary waves on a string with length, tension and mass per unit length of the string
2. Investigation of interference effects to include the Young's slit experiment and interference by a diffraction grating
3. Determination of g by a free-fall method
4. Determination of the Young modulus by a simple method
5. Determination of resistivity of a wire using a micrometer, ammeter and voltmeter
6. Investigation of the emf and internal resistance of electric cells and batteries by measuring the variation of the terminal pd of the cell with current in it
7. Investigation into simple harmonic motion using a mass–spring system and a simple pendulum
8. Investigation of Boyle's (constant-temperature) law and Charles's (constant-pressure) law for a gas
9. Investigation of the charge and discharge of capacitors; analysis techniques should include log-linear plotting, leading to a determination of the time constant RC
10. Investigate the relationship between magnetic flux density, current and length of wire using a top-pan balance
11. Investigate the effect on magnetic flux density of varying the angle using a search coil and oscilloscope
12. Investigation of the inverse-square law for gamma radiation

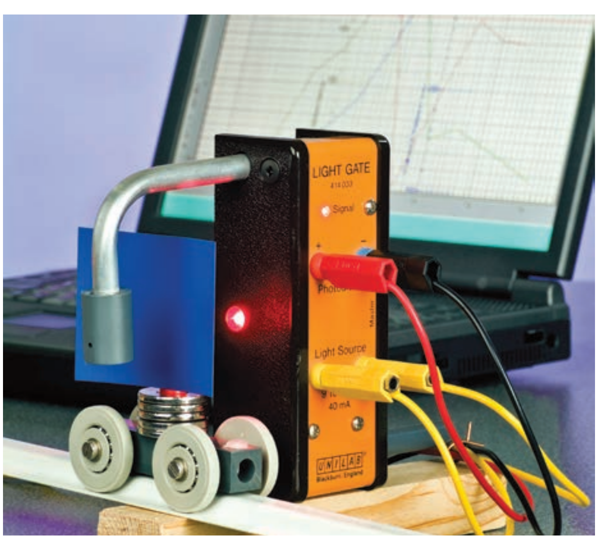
Information about the apparatus, techniques and analysis of required practicals 1 to 6 are found in the relevant chapters of this book, and information on required practicals 7 to 12 will be given in Book 2.

You will be asked some questions in your written examinations about these required practicals.

Practical skills are really important. Take time and care to learn, practise and use them.



An oscilloscope



A motion experiment using a light gate

1 MEASURING THE UNIVERSE

PRIOR KNOWLEDGE

You will have carried out experiments and made measurements in your previous studies of science, so you will know something about the scientific method.

LEARNING OBJECTIVES

In this chapter you will find out how to get a rough idea of atomic size by a simple experiment. You will learn about physics experiments and measurements in general: what units to use and how they are defined; how errors can occur; and how to estimate the uncertainty in your experimental results.

(Specification 3.1.1, 3.1.2, 3.1.3, 3.2.1.1 part)

One of the big questions in physics is: “What is the Universe made of?” Until 1998, most physicists would have said “matter and energy” and been reasonably confident what that meant. “How much matter and energy?” seemed the more pertinent question (Figure 1). Albert Einstein had shown that mass and energy are interchangeable. Their combined amount determines the ‘energy density’ of our Universe, a quantity that will decide its ultimate fate.

In 1929 Edwin Hubble published measurements that showed that the Universe was expanding. Physicists knew that gravity would act to slow the rate of expansion. If the energy density was low, then the Universe would keep expanding, but at a slower and slower rate. A high value of energy density would eventually stop the expansion, and the Universe would begin to contract, eventually ending in a ‘Big Crunch’.

In 1998 observations of distant supernovae showed that the expansion was not slowing at all, but speeding up. The measurements were reproduced by independent research teams, some using different methods. These results suggested that something unknown must be pushing the Universe apart. This is now called ‘dark energy’ and it seems to make up almost 70% of the Universe. Much of the remaining 30% is also mysterious. Work by Vera Rubin in the 1970s on the rotation of galaxies had shown that there must be a significant amount of mass in the Universe that we cannot observe – now known as ‘dark matter’. Current thinking is that the Universe contains a mere 4.9% ‘ordinary’ matter and energy, and researchers are aiming to discover what the mysterious ‘dark’ quantities might be.

Physicists are often labelled either theoretical or practical. Einstein was firmly in the theoretical camp. Saul Perlmutter shared the 2011 Nobel Prize in Physics for practical measurements of the expansion rate of the Universe. Both aspects of physics are equally important. As Robert Millikan (Nobel Prize winner in 1923 for his work on the elementary charge) put it:

“Physics walks forward on two feet, namely theory and experiment. … Sometimes it is one foot first; sometimes the other, but continuous progress is only made by the use of both.”

Figure 1 *(background) How much matter and energy is there in the Universe? This Hubble Ultra Deep Field view taken by the Hubble Space Telescope shows a vast number of distant galaxies.*

1.1 MEASUREMENT IN PHYSICS

Towards the end of the 20th century, just before the discoveries of dark matter and dark energy, it was suggested that the ‘big questions’ in physics had been answered. In a remarkably similar way 100 years earlier, some eminent physicists felt that physics was almost complete. Newton’s laws described forces and motion, Faraday had linked electricity and magnetism, and Maxwell’s equations described electromagnetic waves. Michelson, famous for measurements of the speed of light, went as far as to say:

“The more important fundamental laws and facts of physical science have all been discovered, the possibility of their ever being supplanted (by) new discoveries is exceedingly remote.”

But then, as now, physics was turned upside down by experimental discoveries. Radioactive decay, for example, proved hard to explain for 19th-century scientists, who were still arguing about whether atoms really existed. Observations, measurements and the analysis of recorded data provide the basis for discoveries and advancement in physics.

1.2 THE SCALE OF THINGS

Scientific notation

Physicists investigate matter and energy in the Universe on every scale, from infinitesimally small measurements of subatomic particles to inordinately large ones, like galaxies. These measurements generate very large and very small numbers. We need a concise way of writing the numbers, to avoid strings of zeros across the page. Large numbers are written as a number from 1 to 10, multiplied by a power of 10. For example, the speed of light, $c = 300\,000\,000\,\mathrm{m\,s^{-1}}$, can be written as $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. In a similar way, small numbers are written as a number between 1 and 10, multiplied by a negative power of 10. In this way the wavelength of red light, $\lambda = 0.000\,000\,650\,\mathrm{m}$, would be written as $\lambda = 6.50 \times 10^{-7}\,\mathrm{m}$. This method of writing large or small numbers is known as **scientific notation**, often referred to as **standard form** in the UK.

It is usual to use powers of 10 that go up in steps of 1000, or 10^3, so the wavelength of the red light would probably be written as $650 \times 10^{-9}\,\mathrm{m}$. When using SI units (Système International d’Unités) (see the next subsection), the powers of 10^3, 10^6, 10^9 and so on are given names, such as kilo or mega. These have abbreviations used as prefixes, so a distance of 1000 m (10^3 m) is known as a kilometre and is written 1 km. The names and prefixes that you may come across at AS and A-level are shown in Table 1.

QUESTIONS

1. Satellite TV signals are transmitted on a frequency of 27 000 000 Hz. Rewrite this number using scientific notation.
2. The mean distance from the Earth to the Sun is about 149 600 000 km. Rewrite this in scientific notation. (*Careful!* The distance is given in km in the question.)
3. How long does it take light to travel across the room you are in? (Distance = speed × time, speed of light = $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.)

Multiplication factor		Prefix	Symbol	Example length
1000 000 000 000	10^{12}	tera	T	Radius of Pluto’s orbit (5.9 Tm)
1000 000 000	10^9	giga	G	Mean Earth–Moon distance (0.4 Gm)
1000 000	10^6	mega	M	Mean radius of Earth (6.37 Mm)
1000	10^3	kilo	k	Distance from Manchester to London (320 km)
0.001	10^{-3}	milli	m	Microwave wavelength (~mm)
0.000 001	10^{-6}	micro	μ	Wavelength of visible light (~μm)
0.000 000 001	10^{-9}	nano	n	Approximate atomic diameter (~nm)
0.000 000 000 001	10^{-12}	pico	p	Wavelength of a gamma ray (~pm)
0.000 000 000 000 001	10^{-15}	femto	f	Approximate diameter of an atomic nucleus (~fm)
0.000 000 000 000 000 001	10^{-18}	atto	a	Range of weak nuclear force (~am)

Table 1 *SI prefixes and symbols*

Choosing the units

Physics describes the world in terms of the values of physical quantities. A physical quantity is something that can be measured, such as speed, energy or mass. Each measurement needs a unit, a standard value that is well defined. Giving your height as '1.65' means nothing, but giving it as 1.65 metres makes it clear. The numerical value of a measurement depends on the unit that is used (Figure 2).

Figure 2 *Your mass might be 56, 8.8, 123 or 1.1, depending on whether you measured it in kilograms, stones, pounds or hundredweight. British school science lessons have not used imperial units (feet, stones, pounds and so on) since the early 1970s, but most students still give their height in feet and inches.*

Units were often chosen in the past to be a convenient size for the measurement, like the 'grain' (64.8 mg) traditionally used for the mass of medicines and gunpowder, or the carat (200 mg) used for precious stones. But it is awkward to have many different units for each physical quantity. Every unit has to be defined in terms of a standard, so that the entire world can agree on its magnitude. It is not feasible to maintain several standard definitions for each physical quantity like length or mass, and there is the problem of converting between units, which has been known to have disastrous results (Figure 3).

The Système International d'Unités (**SI**) for defining units of measurement was established in 1960 and is now almost universally accepted – by the scientific world at least. SI units are defined in terms of seven **base units**. These are shown in Table 2. All other units, known as **derived units**, can be defined in terms of these base units.

Figure 3 *In 1999, the Mars Climate Orbiter probe was destroyed because one of the control systems used imperial units (feet and inches), but the navigation software used metric (SI) units.*

Base quantity	Name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 2 *SI base units*

The base units are now almost all defined in terms of physical constants. For example, a length of one metre is defined in terms of the speed of light:

One metre is the length of the path travelled by light in vacuum during a time interval of $\frac{1}{299\,792\,458}$ of a second.

This rather arbitrary time is chosen to match the older definition of the metre. This modern definition of length depends on specialised equipment, but in principle every country can have the same standard metre. However, we also need an independent definition of the second.

> One second is the time taken for 9 192 631 770 complete oscillations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

The atomic clocks on the satellites that make up the Global Positioning System (GPS) (Figure 4) are stable

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to 1 part in 10^{12}; in other words, they are accurate to within 1 second in 32 000 years.

The only base unit not yet defined in terms of a universal constant is the kilogram. This is still defined as the mass of a particular cylinder of platinum–iridium alloy, the International Prototype Kilogram (IPK), which is kept in a vault in Paris.

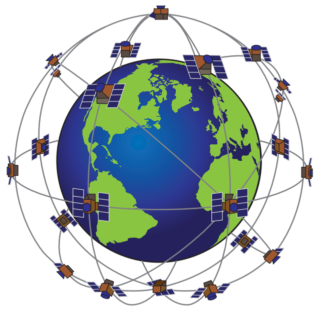


Figure 4 *To use the GPS system, the receiver must be able to see a minimum of four satellites.*

The kilogram is also the only base unit with kilo in its name. Logically, the base unit of mass should be the gram, but a historical quirk meant that the kilogram was chosen. This is important for calculations. If you need to put a value of mass into an equation, you must use kilograms, so a mass of one kilogram = 1 kg, whereas a mass of one gram = 1×10^{-3} kg.

QUESTIONS

4. The IPK is kept in a controlled atmosphere, and is only rarely taken from its vault. Why?

5. The IPK is a right-circular cylinder (height = diameter) of 39.17 mm. Why is it this shape? Why is the choice of metal important?

6. Use the formula density = $\dfrac{\text{mass}}{\text{volume}}$ to find the density of the standard (IPK) kilogram. The SI derived unit for density is kg m^{-3}. Remember that 1 mm = 1×10^{-3} m, so 1 mm^3 = 1×10^{-9} m^3.

7. The earliest units for length were based on the human body, for example the cubit in ancient Egypt was defined as the distance from the tip of the forefinger to the elbow. Give an advantage, and a disadvantage, of this system.

8. The speed of light is now exactly 299 792 458 m s^{-1}. Why 'now' and why 'exactly'?

Units for all the other physical quantities, such as velocity, acceleration, force and energy, are derived from these base units. For example, the unit for velocity is metre per second (m s^{-1}). Derived units are sometimes given names, like newton (N) for force, and joule (J) for energy. As a rule the named unit is written in full with a lowercase initial letter, but the abbreviation begins with an uppercase letter. For example, the SI unit for frequency is the hertz, abbreviated to Hz.

All named derived units can be expressed in terms of the base units. For example, one newton is defined as the force that will accelerate a mass of one kilogram by one metre per second, every second. This definition can be represented by the equation $F = ma$.

As the units have to be the same on both sides of this, or any other, equation (you could not have metres = kilograms, for example), then, in terms of units, $F = ma$ becomes

$$1\,\text{N} = 1\,\text{kg} \times \frac{1\,\text{m}}{(1\,\text{s} \times 1\,\text{s})} = 1\,\text{kg}\,\text{m}\,\text{s}^{-2}$$

QUESTIONS

9. One pascal (1 Pa) is the SI derived unit of pressure. Since pressure = $\dfrac{\text{force}}{\text{area}}$, write 1 Pa in terms of SI base units.

10. Express the joule in terms of base units. *Hint:* What equation links energy or work to other quantities?

Physicists do not always quite stick to the rules for using SI units. Sometimes it is just too clumsy to use the SI base unit. For example, the kilogram is rather large when it comes to the mass of an atom, so the atomic mass unit is used. The metre is too small for interstellar or intergalactic distances, so astronomers use light years or megaparsecs. You might have used kilowatt hours, rather than joules, to measure the electrical energy used in a house. It is of course possible to convert all these to the relevant SI unit.

In the assignment you can find a value for the size of an atom in metres.

ASSIGNMENT 1: FINDING A MAXIMUM SIZE FOR AN ATOM

(MS 0.1, MS 0.2, MS 2.3, MS 4.3, PS 1.1, PS 3.2)

There is a way to find a rough value for the size of an atom using ordinary school laboratory equipment. Olive oil, a very clean tray, a magnifying glass and scale, a ruler and some fine powder, such as lycopodium (that is, pollen – a potential allergen) are needed.

The general idea is to let a small drop of olive oil fall onto the surface of some water (Figure A1). The drop will spread out into a very thin film. If the surface of the water is coated with powder first, it will allow the oil film to be seen more clearly.

The volume of the film will be the same as the volume of oil in the drop. In theory the film of oil should be a circle. (In practice it is not!) Imagine that the film is roughly cylindrical in shape (a very thin cylinder). Then if the volume and the area of the film are known, its thickness can be calculated. The oil molecules cannot be bigger than this thickness, and an atom must be smaller still, so we can arrive at the maximum size for an atom.

Figure A2 is a scale drawing of typical results. If you are able to do this experiment yourself, you can use your own results. Otherwise, use measurements from Figure A2.

Questions

A1 Find the volume of the drop (volume of sphere = $\frac{4}{3}\pi r^3$).

A2 Find the area of the film (area of circle = πR^2).

A3 Find the thickness of the film, h.

A4 An olive oil molecule is 10 atoms long. What is the maximum diameter for an atom in metres?

A5 What have you assumed in this calculation?

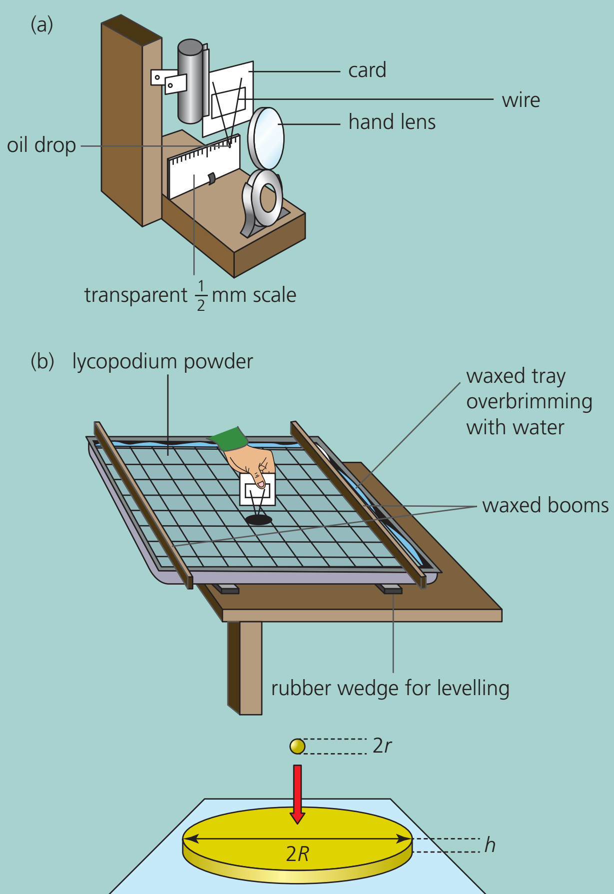


Figure A1 *The oil drop experiment*

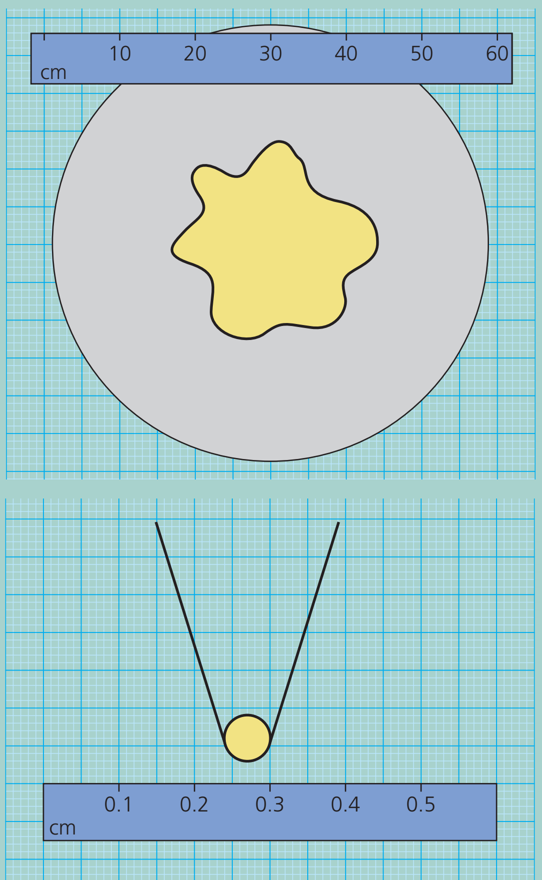


Figure A2 *Volume of the drop = volume of the oil film = area of the oil film × thickness of the film*

KEY IDEAS

- We use SI units. There are seven base units. All other units are derived from these.
- Large and small numbers are expressed in standard form, for example 3.0×10^8.
- Multiples of units in powers of 10 increasing in threes, for example, 10^3, 10^6, and so on, are given standard prefixes, for example, kilo (k), mega (M).

1.3 EXPERIMENTS IN PHYSICS

Experimental error

It is surprising that we can get an estimate for the size of an atom using such simple apparatus as that used in Assignment 1. After all, we have managed to measure something that is far too small to see. How do we know that our answer is right? What errors might we have made? Can we correct them, reduce them or at least account for them?

The term 'experimental errors' does not generally refer to the sort of blunders we all make from time to time, such as forgetting to connect the battery, misreading a scale or failing to take a reading at the right time. These are annoying, but repeating the experiment with more care usually solves the problem. Experimental errors fall into two types: random errors and systematic errors.

Random errors can cause readings to be too high or too low. Just as the name suggests, the readings fluctuate about the mean. Random errors may arise due to a number of different causes.

- Observation or reading errors: perhaps when timing the oscillations of a pendulum, or when trying to read the flickering needle on the dial of an analogue meter.
- Environmental: perhaps the temperature of the room is fluctuating, or the supply voltage keeps changing.

The crucial thing about a random error is that it is equally likely to give you a result that is too high as one that is too low. Repeating the readings and calculating a mean value is useful because the more readings you have, the more the random fluctuations will be averaged out.

Systematic errors on the other hand lead to results that are consistently wrong. Repeating these readings is pointless, since the error occurs in the same way each time. Systematic errors may also occur due to a number of reasons.

- Instrument error: a poorly calibrated thermometer, for example, or a top-pan balance that has not been zeroed correctly.
- Reading error: perhaps due to parallax error (Figure 5) when reading the scale.
- Poor experimental design: for example, ignoring the effect of an external factor like magnetic field, temperature or pressure.

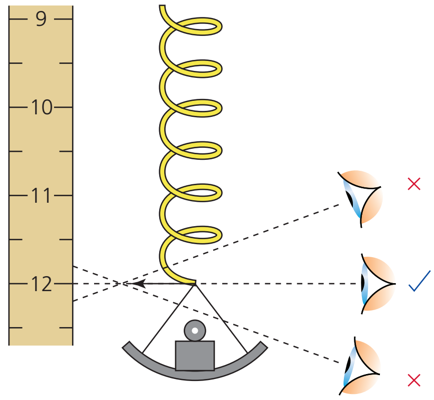


Figure 5 *Try to position your eye close to the scale and look in the correct direction to avoid parallax errors.*

Ernest Rutherford (Figure 6), who discovered the atomic nucleus, believed that experiments should have a clear outcome:

"If your experiment needs statistics, you ought to have done a better experiment."

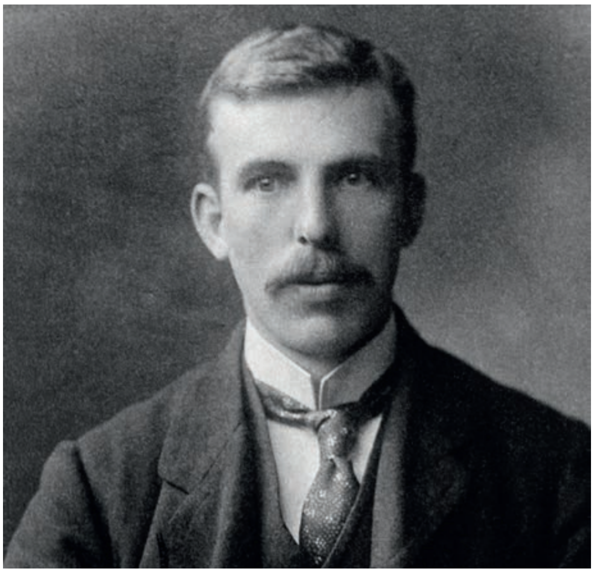


Figure 6 *Ernest Rutherford*

QUESTIONS

11. a. You intend to use a top-pan balance to find the mass of a particular ball bearing. Is it worth repeating the measurement several times and taking an average?

    b. Suppose you need to find the mass of a typical ball bearing. How would you make your result as accurate as possible?

12. When you are measuring the diameter of a wire, it is good practice to take readings at several points along the length of the wire. The readings should also be taken in different orientations. Explain why.

Accuracy, precision and uncertainty

How sure of our measurements can we be? This can be a difficult question to answer, especially if you happen to be one of the first to make the measurement. In practice, the experiment is repeated by the experimenter to check that it gives consistent results. If so, then the measurement is said to be **repeatable**. If other experimenters get similar results, preferably in different laboratories using different techniques, then the measurement is said to be **reproducible**.

A result is said to be **accurate** if it is close to the true value, that is, the standard or accepted value. In exceptional cases, of course, the new measurement may not agree with the accepted value, because the accepted value is wrong. However, you need to be very sure before making a claim like that. The standard 'textbook' answer will have been repeated many times, probably in different laboratories and using different methods. If the new results prove to be repeatable and reproducible, it may mean that an established theory could be wrong (Figure 7).

'Precision' does not mean that the measurements are right; it merely tells you whether the results are numerically close together. For example, suppose that a measurement was made five times and the results were 3.223, 3.222, 3.223, 3.221 and 3.223. These results vary through a range

Figure 7 *In 2011 physicists at the Gran Sasso laboratory in Italy measured the speed of neutrinos emitted by the accelerator at CERN and found they were travelling faster than the speed of light. Special relativity says it is impossible for a particle to reach the speed of light, as this would give it an infinite mass. Faster-than-light travel also raises the possibility of time travel. Physics Professor Jim Al-Khalili promised to eat his boxer shorts on live TV if the measurements were shown to be right. In fact, the measurements were wrong … they were caused by a loose fibre-optic cable!*

of 0.002, from the lowest to the highest value recorded. The mean of the five readings is 3.222 (correct to four significant figures), so we can say that the **uncertainty** in this mean value due to the scatter of results is ± 0.001. This seems a small uncertainty but it depends on the size of the measurement. We need to compare this uncertainty in the readings with the overall result by expressing it as a percentage. The percentage uncertainty is $(0.001/3.222) \times 100\% = 0.031\%$. This is a very small percentage uncertainty, so the results could be said to be very **precise**. That does not mean that the results are correct or even accurate. They could all be wrong in the same way. Figure 8 gives a visual summary of the meanings of precision and accuracy.

When recording your results, you should be careful not to overstate the precision by writing an excessive number of digits in your answer. Suppose that three teachers have timed the school 100 m sprint race and have recorded times of 12.3, 12.5 and 12.6 s for the winner. The average time was 12.466 66 s. But each teacher's reading had an uncertainty of 0.1 s at least, and the range of their readings was 0.3 s, or ± 0.15 s, so the result

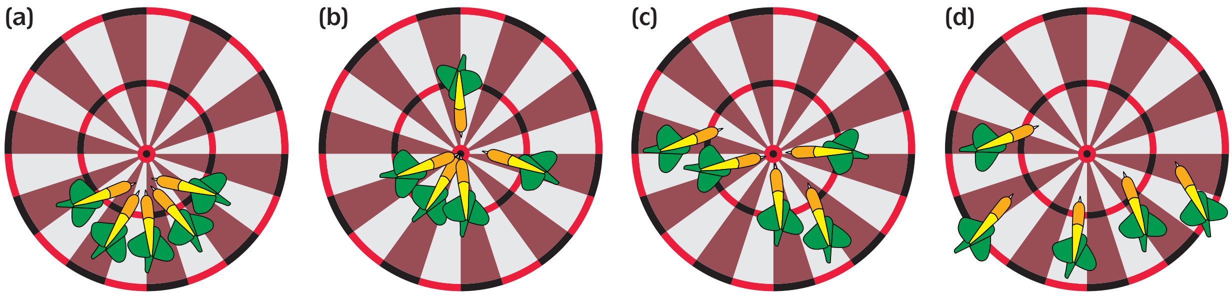


Figure 8 *Precision and accuracy: (a) precise but not accurate; (b) precise and accurate; (c) accurate but not precise; (d) neither accurate nor precise.*

should be quoted to a similar level of precision: (12.5 ± 0.15) s is more reasonable but, since we usually err on the side of caution, (12.5 ± 0.2) s is probably appropriate.

On a similar theme, it would be a mistake to record this set of readings, all taken with the same equipment:

Time T / s	1.214	1.20	0.800	0.5

If the readings are all made with the same precision, they should all be quoted to the same number of **significant figures**. The trailing zeros are important!

QUESTIONS

13. a. Describe a real situation where measurements could be precise, but not accurate.

b. Describe a real situation where measurements could be accurate, but not precise.

Some readings do not vary widely but are still not precise, simply because they are measured with a device with low **resolution**. The resolution of a measuring device is the smallest increment in the measured quantity that can be shown on the device. For example, you could find the mass of an object using a digital balance with a resolution of (that is, gives readings in) grams, tenths of grams, or hundredths of grams. Suppose you were using the balance that measured to the nearest gram to find the mass of a mango; your readings could be out by 0.5 g. In fact, it is worse than this because when you zeroed the balance, it could also have been out by 0.5 g. So your reading is said to have an uncertainty of ±1 g. As a rule of thumb you can estimate the uncertainty associated with taking a reading to be ± the smallest scale division.

Measurements should always be written with a value, the associated uncertainty and an appropriate unit, for example, mass of a mango = (142.3 ± 0.1) g. This is not too much of a problem, with less than 0.1% uncertainty. But if you were finding the mass of a blackcurrant, you would probably want a balance with a higher resolution.

You can improve the precision of measurement of repeatable events by just doing a lot of them. Galileo is said to have timed the oscillations of a pendulum (candelabras hung from the ceiling of a church) using his pulse as a time keeper. This is not a very high-resolution instrument. But by timing 10 oscillations he could share the uncertainty among all 10 oscillations, and arrive at a more precise answer.

Whether you are measuring the speed of neutrinos, or the area of an oil film, it is important to know how precise your result is. Every experimental result should be accompanied by an estimate of its uncertainty. For example, the currently accepted value for the mass of an alpha particle is given as

$$(6.644\,656\,75 \pm 0.000\,000\,29) \times 10^{-27}\ \text{kg}$$

sometimes written as

$$6.644\,656\,75(29) \times 10^{-27}\ \text{kg}$$

The numbers in brackets indicate the uncertainty in the last two digits. That is a high-precision measurement, an uncertainty of 29/664 465 675, or less than 5×10^{-6}%. That is equivalent to knowing the distance from London to New York to within 25 cm.

QUESTIONS

14. Estimate the resolution of Galileo's time keeper. Suppose the candelabra took 2 s to complete one oscillation. What would Galileo's result be if he timed one oscillation? What would the uncertainty be? What would the percentage uncertainty be? How would these answers be changed if he now timed 10 oscillations and used that to calculate the time for one oscillation?
15. If you did not have a high-resolution balance, how could you find the mass of a blackcurrant more precisely? (Assume that you have a large number of blackcurrants to hand!)
16. Suppose you took three oranges and found their mean mass, using a balance with a resolution of 0.1 g. Why would it be wrong (and certainly misleading) to write your answer as 121.333 333 g? How would you write it?
17. a. What is the uncertainty associated with measuring the width of this book? (Suppose that you used a 30 cm ruler with mm divisions.) Write your answer as result ± uncertainty, followed by the correct unit.
    b. Why is the uncertainty more of a problem if I asked you to use the same ruler to measure the thickness of the book?
    c. How would you find the thickness of one page of this book? How precise do you think you could be?

KEY IDEAS

- Experimental errors can be systematic. These tend to affect all readings in the same way. Repeating the readings does not help. Try to improve the method.
- Experimental errors can be random. These fluctuate above and below the mean value. Repeated readings will improve the precision of these readings.
- No measurement of a physical quantity is ever exact. There is always an uncertainty associated with it.
- An accurate measurement is one that is close to the accepted value.
- A precise set of measurements are closely grouped together, with little spread or uncertainty.

1.4 COMBINING UNCERTAINTIES

The final result of an experiment is often a combination of several measurements. That means that the overall uncertainty will depend on a combination of the precision of each measurement.

What if you are *adding or subtracting* two quantities? The general rule is:

If you are adding or subtracting quantities, you need to add their absolute uncertainties.

An absolute uncertainty is the possible deviation from the mean value in the unit of measurement. Finding the difference of two measurements can lead to large *percentage* uncertainties.

Worked example 1

In a situation like that in Figure 9, we might have these readings:

Reading A : Mass of bowl = (200 ± 1) g, which is a percentage uncertainty of 0.5%.

Reading B: Mass of bowl *and* flour = (220 ± 1) g, a percentage uncertainty of ≈0.5%.

Mass of flour = reading B − reading A = (20 ± 2) g

The uncertainty in this difference between two measured values is 2 g since reading B might be higher by 1 g and reading A might be lower by 1 g, and vice versa. The percentage uncertainty is now 10% (compared with 0.5% in the measured values).

Figure 9 *Finding a difference in two quantities, for example finding the mass of sugar in a bowl, can lead to large percentage uncertainties.*

What if you are *dividing or multiplying* two quantities? Suppose you have been given a small metal cube, which you suspect is made of lead. You might decide to measure the density of the cube to see if it could indeed be lead. The density of a material is defined as the mass of a given volume. In SI units this should be measured in kilograms per cubic metre, but grams per cubic centimetre is also commonly used. Lead has a density of $11.34\,\mathrm{g\,cm^{-3}}$.

As density equals mass divided by volume,

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

you might begin by using a top-pan balance to measure the mass, and a ruler marked in millimetres to measure the dimensions of the cube.

Mass of metal cube = (89 ± 1) g

Length of metal cube = (2.1 ± 0.1) cm

Width of metal cube = (1.9 ± 0.1) cm

Depth of metal cube = (2.1 ± 0.1) cm

The volume of the cube $= 2.1 \times 1.9 \times 2.1 = 8.379$ cm^3

But how precise is this measurement? It is possible that all the dimensions have been underestimated by 0.1 cm, so the volume could be as large as $2.2 \times 2.0 \times 2.2 = 9.68$ cm^3. Similarly, the volume could be as small as $2.0 \times 1.8 \times 2.0 = 7.20$ cm^3. Possible values for the volume of the cube are from 7.20 to 9.68 cm^3, a range of 2.48 cm^3, so the uncertainty is approximately ± 1.24 cm^3. The volume of the cube is therefore (8.38 ± 1.24) cm^3, an uncertainty of almost 5%.

It is possible to find the uncertainties in calculated values by inserting the largest and smallest values of your data into the relevant formulae. But this can be time-consuming and there is a better way. You saw in Worked example 1 that to find the uncertainty in the difference of two masses you simply add the individual uncertainties together. But if you are *multiplying* or *dividing* two quantities, the general rule is:

If you are multiplying or dividing quantities, then you add the *percentage uncertainties* together.

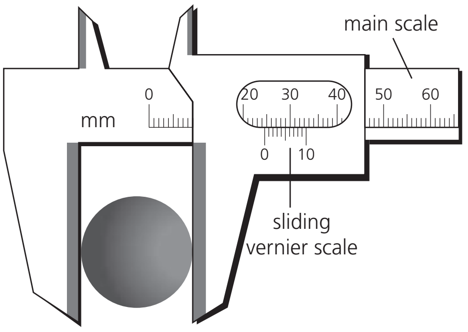
Worked example 2

Using the data given above for the metal cube, the percentage uncertainties in each measurement of length are $0.1/2.0 = 5\%$. So the uncertainty in volume is $5 + 5 + 5 = 15\%$.

To find the density, we need to divide the mass by the volume. The percentage uncertainty in the measurement of mass $= 1/89 = 1.1\%$. The overall uncertainty in the density value is therefore $15 + 1.1 = 16\%$.

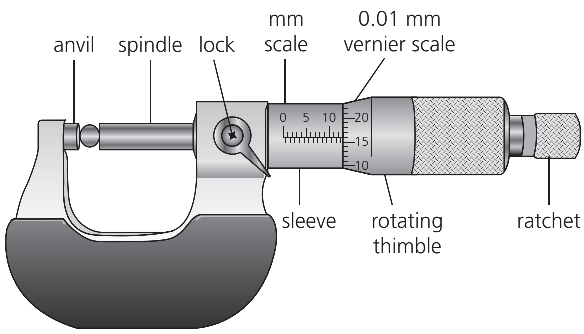
Density of the cube $= 89/8.37 = 10.6 \pm 16\%$ or (10.6 ± 1.7) g cm^{-3}.

Since the accepted value for the density of lead, 11.34 g cm^{-3}, falls within the range of uncertainty, the metal could be lead. The measured value for density is not precise enough, however, to rule out other metals. We could improve the precision by using instruments with better resolution to measure length and mass. Two instruments commonly used in the laboratory to measure length precisely are shown in Figures 10 and 11. They both use a **vernier scale** – a movable scale that allows a fractional part on the main scale to be determined.



The reading in mm is taken from the position of the zero on the sliding scale. Here this is between 24 and 25. The next significant figure (to 0.1 mm) is found by judging which scale mark on the sliding scale is perfectly aligned with a mark on the main scale. Here this is 5.
The reading is 24 + 0.5 = 24.5 mm.

Figure 10 *Vernier callipers can measure length to one-tenth of a millimetre.*



Turning the ratchet moves the spindle until it just touches the object. The ratchet then slips to avoid deforming the object.

The reading to the nearest 0.5 mm is taken where the thimble meets the sleeve. Here this is 12.5 mm. The final significant figures are given by judging which mark on the rotating scale coincides with the horizontal line on the sleeve. Here this is 16.

The reading is 12.5 + 0.16 = 12.66 mm.

Figure 11 *A micrometer screw gauge can measure length to one-hundredth of a millimetre.*

KEY IDEAS

- If two physical quantities are to be added or subtracted, then their uncertainties must be added.
- If two physical quantities are to be multiplied or divided, then their percentage uncertainties must be added.

QUESTIONS

18. Suppose you could improve the precision of either the measurement of length or the measurement of mass, for the small metal cube considered in the text. Which would most improve your final answer?

19. The cube may not be perfect, so that the dimensions may differ at different points. How would you allow for this?

20. You have been asked to find the density of a liquid, which you suspect is ethanol, which has a density of around 80% that of water. Suppose that you measure the volume using a measuring cylinder and its mass on a top-pan balance. By estimating the values of the mass and volume of ethanol you would use, and the resolution of the measuring instrument, calculate an approximate value for the uncertainty in your answer.

ASSIGNMENT 2: FINDING THE UNCERTAINTY IN THE ATOMIC DIAMETER MEASUREMENT

(MS 0.4, MS 1.1, MS 1.5, PS 1.1, PS 2.1, PS 2.3, PS 3.2, PS 3.3)

It would be useful to estimate the uncertainty in our measurement of atomic size in Assignment 1.

- First you need to estimate the uncertainty in all your measurements.
- Then calculate the percentage uncertainty for your readings.
- Estimating the uncertainty in the area of the oil film is difficult. One way would be to estimate the largest and smallest area that the film could be. This will give you a spread of results. Halve this to find the uncertainty.
- Combine the uncertainties.

For example, the uncertainty in the diameter of the oil drop could be ±0.1 mm. This is a significant uncertainty, since the drop only has a diameter of 0.5 mm, so that is a ±20% uncertainty. The radius = (0.25 ± 0.05) mm since we divide the uncertainty by 2 as well. But the uncertainty in the volume will be larger than that because volume depends on the radius cubed, $V \propto r^3$, so that is three times the percentage uncertainty. In this case volume has a percentage uncertainty of 60%.

Questions

A1 What is the final uncertainty in your value for atomic diameter?

A2 How would you improve the experimental method to try to reduce the uncertainty in this answer?

1.5 USING GRAPHS

A common way of reducing the uncertainty in a measured quantity is to repeat the reading, using a set of different values of the independent variable, and then plot a graph. Suppose you were asked to find the mass of a raindrop. You have an electronic top-pan balance with resolution of 0.01 g. Assume that you can count the raindrops! You could use any one of the following methods:

A Catch one raindrop and find its mass.

B Repeat the above method lots of times and find the mean mass.

C Collect 100 drops, find the mass and divide by 100.

D Collect 100 drops, recording the mass after every 10 drops. Then plot a graph of your answers.

Which method will give the most precise, and the most useful, results?

Method A will give a very large percentage uncertainty. The average mass of a raindrop depends on the type of rain (it varies from mist to downpour!) but is unlikely to be much more than 100 mg. The reading would have a percentage uncertainty of

$$\left(\frac{0.01}{0.1}\right) \times 100 = 10\%$$

Method B is better, but tedious! Theoretically, the precision is equivalent to that of method C, which would give a percentage uncertainty of

$$\left(\frac{0.01}{10}\right) \times 100 = 0.1\%$$

In practice, drying the container between each drop would be ridiculous. Method D gives the same uncertainty as method C, but allows you to spot any results that do not fit the pattern and ignore them if they are genuinely anomalous results. The results of such an experiment are shown in Table 3 and the graph obtained is shown in Figure 12.

Number of raindrops	Accumulated mass / g
10	3.60
20	3.70
30	3.80
40	3.90
50	4.01
60	4.11
70	4.20
80	4.30
90	4.40
100	4.49

Table 3 *Mass of every 10 raindrops*

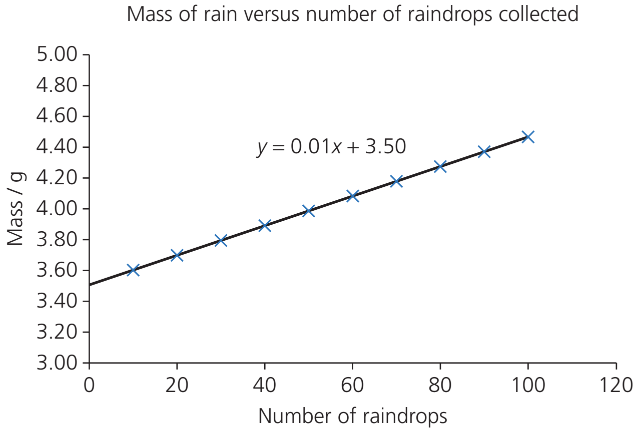


Figure 12 *The equation of a straight line is always of the form $y = mx + c$, where y is the variable plotted vertically, x is the variable plotted horizontally, m is the gradient and c is the intercept on the y-axis. If the equation is to be a straight line, m must be a constant (fixed number). In this case the equation says mass collected (y) = average mass of a raindrop (m) × number of raindrops (x) + any other mass (perhaps the container or a zero error on the balance).*

The gradient of the line is found by calculating $\frac{\text{difference in } y}{\text{difference in } x}$, which in this case is

$$\frac{\text{mass}}{\text{number of drops}} = \text{mass of one raindrop}$$

The intercept of the line on the y-axis gives a mass reading before any raindrops are collected. This value, 3.50 g in this case, is a zero error, which might not be noticed without the graph.

QUESTIONS

21. Look at the graph in Figure 13. It shows another set of results from the raindrop experiment. What do you think happened? Could you still use the results?

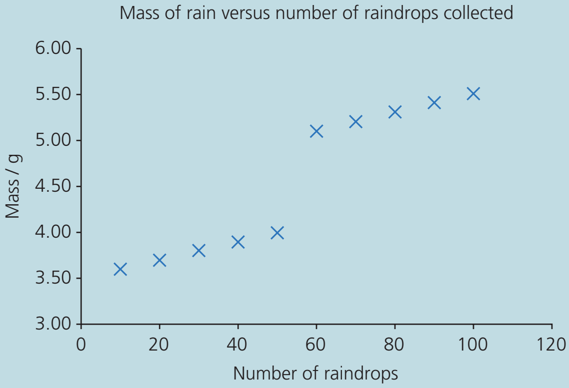


Figure 13 *Plot of another set of results*

Plotting graphs of experimental results

Graph-plotting in physics is not quite the same as in mathematics, where you are often plotting a function with perfectly accurate points that lie on an ideal curve. Physics data is often taken from real-life experiments and has some uncertainty associated with it. The data points will be scattered rather than being a perfect fit to a function. We often do not know whether the results are following a mathematical law or not. Indeed, that is often what we are trying to find out. In practice, it is difficult to draw quantitative conclusions from a curve, so we try to draw straight-line graphs to test relationships between quantities. This may mean that we plot a function of the variables, for example, x^2 or $1/x$, instead of the raw data.

Suppose that you were studying a falling object (Figure 14) and took a series of measurements

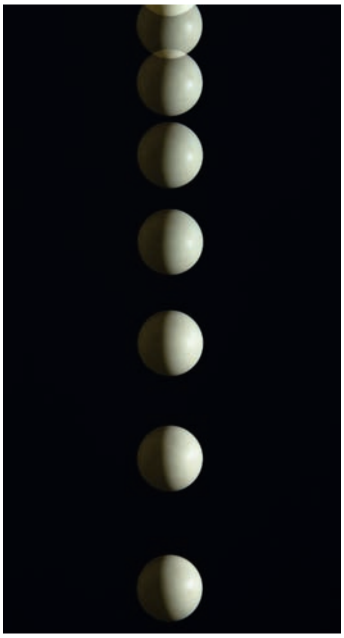


Figure 14 *Time-lapse image of a falling object*

showing how far the object had fallen after certain periods of time, say after 1, 2, 3, … seconds. You would get a graph like the one shown in Figure 15(a). It may *look* as if distance depends on time squared but we cannot be sure from a curved graph. The distance fallen, s, and the time taken, t, could be related by a quadratic (squared) equation like $s = At^2 + B$, where A and B are constants (just a fixed number). This needs to be compared with the equation of a straight line, $y = mx + c$:

$$\begin{array}{ccccc} s & = & At^2 & + & B \\ \downarrow & & \downarrow\downarrow & & \downarrow \\ y & = & mx & + & c \end{array}$$

Plotting s on the y-axis and t on the x-axis gives a curve (Figure 15a), but if we plot s on the y-axis and t^2 on the x-axis, we should get a straight line (Figure 15b). If the points are a good fit to the straight line, we can deduce that the experimental data follows the relationship. The gradient will equal the constant A and the y-intercept will equal the constant B.

Table 4 shows a few examples of what to plot in order to confirm a relationship.

Good practice in graph drawing

Accuracy in graph work is important, not least because it often accounts for a significant number of exam marks. So here are a few tips on good practice:

- Choose your scales on each axis so that your data spreads over at least half of the axis. Use a false origin if necessary. You do not need to start the graph at (0, 0), unless you have a measured data point to plot there.
- Use a sharp pencil and a ruler to draw the axes.
- Label each axis with the quantity and unit separated by a solidus (slash) /, for example T^2 / s^2, F / N, λ / m, and so on.

Variables	Constant(s)	Possible relationship	Rearrange to	y	x	Gradient (constant for a straight line)	y-intercept
m, T	k	$T = 2\pi\sqrt{\frac{m}{k}}$	$T^2 = 4\pi^2\left(\frac{m}{k}\right)$	T^2	m	$\frac{4\pi^2}{k}$	0
f, λ	c	$c = f\lambda$	$f = \frac{c}{\lambda}$	f	$\frac{1}{\lambda}$	c	0
V, i	E, r	$V = E - ir$	$V = E - ir$	V	i	$-r$	E
F, r	G, M, m	$F = \frac{GMm}{r^2}$	$F = \frac{GMm}{r^2}$	F	$\frac{1}{r^2}$	GMm	0

Table 4 *Examples of what to plot to confirm a relationship*

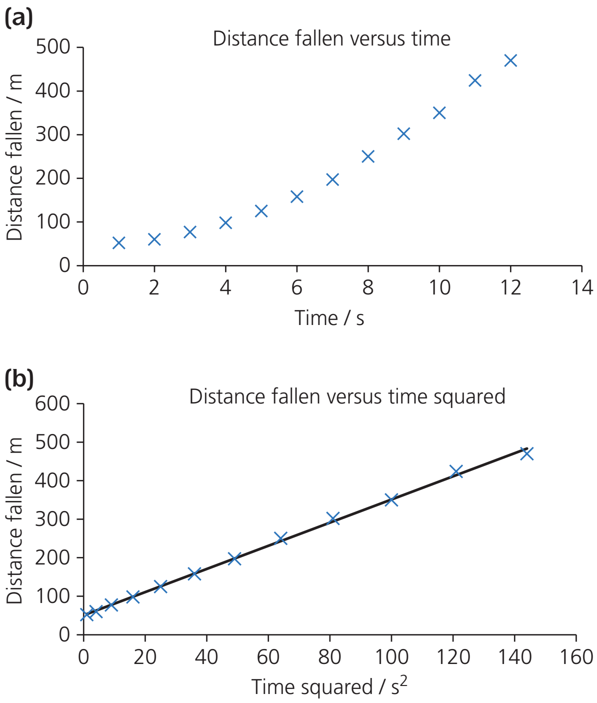


Figure 15 *Finding the relationship between distance fallen and time taken*

- Plot points (using a sharp pencil) with a small cross.
- Give the graph a meaningful title.

Drawing a best-fit straight line and calculating the gradient

If the points look as if they may fall close to a straight line, you may opt to draw a 'best-fit' straight line. When a computer does this mathematically, it chooses the straight line that minimises the total distance of the points from the line. You should aim to do the same. You have two advantages over the computer:

- You can use your discretion and ignore any **outliers**, especially if you have practical reasons to suspect their accuracy. An outlier may pull a computer's best-fit line way off course. Try to identify these anomalous results, repeat them or at least try to explain why they are going to be ignored.
- You may know that the line must go through a given point, (0, 0) for example, and you can pivot your ruler about that point. (Make sure you have a 30 cm clear plastic ruler so that you can see the points through it.)

The gradient of a graph in physics often represents an important physical quantity. For example, if you plot velocity (y-axis) against time (x-axis), the gradient at a particular point gives the value of the acceleration at that time. You will often need to find the gradient of a best-fit line. Choose a large section of the graph, covering at least two-thirds of each axis (Figure 16). This will reduce the effect of any uncertainties in reading the points. Choose your two points, say (x_1, y_1) and (x_2, y_2); then

$$\text{gradient} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

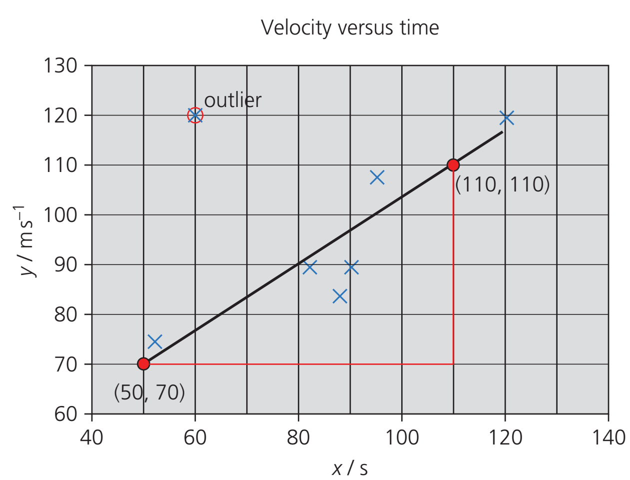
It is useful to include the unit when quoting the value of a gradient. The unit will be that of the quantity on the y-axis divided by that of the quantity on the x-axis. So for a velocity against time graph, the gradient will have a unit of $(\mathrm{m\,s^{-1}})/\mathrm{s} = \mathrm{m\,s^{-2}}$.

Uncertainties in graph plotting

Uncertainties on graphs may arise in two ways:

- There may be a large uncertainty in each measurement.
- It might be difficult to choose the best-fit line to find the gradient.

The first problem can be dealt with using 'error bars'. Plot the reading with a small cross as before. Then use bars through the point to show the horizontal and vertical extent of the uncertainty (see Figure 17).



gradient = $(y_2 - y_1)/(x_2 - x_1)$ = (110 – 70)/(110 – 50) = 40/60
= 0.66 m s^{-2}

Figure 16 *Calculating the gradient of the best-fit line*

The second problem can be dealt with by drawing a best-fit line and then a 'worst-case' best-fit line (see Figure 17). Find the gradient and intercept of both lines. Your answer can be quoted as best-fit gradient ± (difference between the values).

KEY IDEAS

- The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is a constant equal to the y-intercept.
- Uncertainty in a data point can be shown on a graph by drawing error bars.
- Best-fit and worst-fit lines can be drawn through the error bars to estimate the uncertainty in the gradient and intercept values.

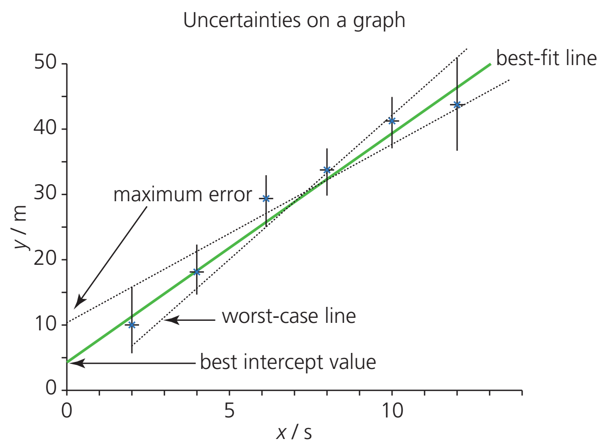


Figure 17 *Best and worst fits through points with error bars*

ASSIGNMENT 3: PLOTTING A GRAPH

(MS 0.1, MS 1.1, MS 1.5, MS 3.1, MS 3.2, MS 3.3, MS 3.4, PS 1.1, PS 2.3, PS 3.1, PS 3.2, PS 3.3)

An experiment has been carried out to measure the time it takes for a pendulum to complete 10 oscillations. Theory suggests that the time for one oscillation, T, depends on the length of the pendulum, l, according to the following equation:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The results are shown in Table A1.

Time for 10 oscillations / s	Length of pendulum / cm
8.7	20.0
12.0	30.2
13.7	40.4
14.2	49.8
14.3	60.2
16.7	70.0
17.0	80.2
19.2	89.9
19.0	100.2
22.0	110.0
25.0	120.0

Table A1

The uncertainty in the time measurement was estimated to be ±0.1 s. The uncertainty in the length was estimated at ±0.1 cm.

Questions

A1 Plot a suitable graph to test the relationship.

A2 Find the gradient of the best and 'worst' case lines.

A3 Find the value of g that this gives you.

A4 Estimate the uncertainty in your answer. Comment on the precision of this result.

A5 The accepted value for g is 9.81 m s^{-2}. Is your result accurate?

1.6 MAKING AN ESTIMATE

Finding a rough value for the diameter of an atom, and the uncertainty in this value, is an example of estimation, helped by a practical measurement or two. Estimation is a very useful skill in physics, and indeed in life!

It is possible to estimate the answers to questions such as: How much water is there in the reservoir? How many people could it supply? How many cars are there on the M25 at a given time? How many wind turbines would be needed to provide all the electricity for a small town? You do not always need an exact answer – sometimes an order of magnitude will do. It is good enough to know whether the answer is tens, hundreds, thousands or millions. An estimate is also useful for checking your answer in an exam question, for example, “Could the radius of Earth really be 6000 m or have I made an error?”

Enrico Fermi, a physicist who built the world’s first nuclear reactor in a squash court, used to ask his students questions like these, which could be solved, very approximately, on the back of an envelope.

- How many grains of sand are there on Earth’s beaches?
- How many piano tuners are there in Chicago?
- How many atoms are in your body?

(This sort of question has also become popular in some university interviews – popular with the interviewers rather than with candidates!)

If you are faced with one of these, such as, “How many cows could you fit in a barn?”:

- Do not panic!
- Think of something you know, or can reasonably guess or find out about the situation.
- Break the problem down into smaller, hopefully easier, questions.
- Make simplifying assumptions, for example treat the cow as a cube!

Fermi gave this example. When asked, “What is the diameter of the Earth?”, he reasoned like this:

1. I pass through three time zones when I fly from New York to Los Angeles.
2. I know that it is about 3000 miles from New York to Los Angeles.
3. That is 1000 miles per time zone, on average.
4. There are 24 hours in a day, so there must be 24 time zones around the world.
5. 24 time zones × 1000 miles per time zone = 24 000 miles.

So that is a circumference of about 24 000 miles. Circumference = $\pi \times$ diameter. Take π as approximately equal to 3, which gives the diameter of the Earth as about 8000 miles. An accurate value is 7926 miles, so 8000 is not a bad estimate!

ASSIGNMENT 4: MAKING AN ESTIMATE

(MS 0.4, MS 1.4)

Here are a few ‘Fermi-type’ questions for you to try. At this stage, the way you approach this is just as important as the result, so record your method as well as your answer. It would be useful to work with a partner, or in a small team, at first so that you can discuss different approaches to the problem. You can look up some of the basic facts if necessary, but try to make as much progress as you can by reasoning from what you know.

Questions

A1 How many Jelly Babies could you fit into a supermarket carrier bag?

A2 “If all the mobile phone chargers in the UK were unplugged when not in use, we could save enough energy to boil 1 000 000 kettles every year.” Could this be true? If it is true, is it important?

A3 In his book *Sustainable Energy*, David MacKay says that trying to save energy by unplugging mobile phone chargers is like “trying to bail out the Titanic with a tea-spoon”. How long would that take?

A4 What mass of plastic is used every year in the UK to hold bottled water?

A5 Make up your own estimation question and swap with another group. (You should have an answer, or at least a way of getting there.)

Moving on: the start of the atomic age

In the first few years of the 20th century, the arguments over atomic reality were quickly forgotten. In 1897 the scientific debate was shifted by two discoveries: that of the electron (by J. J. Thomson in Cambridge) and radioactivity (by Henri Becquerel in Paris). These showed not only that the atom was real, but also that it had a structure and could be taken apart. The search to understand the composition of the atom had begun. But it was not until the 1950s that we could actually see images of atoms – search for 'field ion microscope'.

PRACTICE QUESTIONS

1. You have been asked to measure the thickness of a sheet of printed paper.

a. Describe how you would do this as precisely as possible.

b. Estimate the uncertainty of your reading.

c. The average density of the paper is quoted as 120 g m^{-3}. How would you verify this?

2. The timing of races for a school sports day is done manually. Time keepers for the 100 m race stand at the finish line with stopwatches. They start their stopwatches when they hear the starting pistol and stop them as the runners cross the finish line.

a. The physics teacher points out that the time keepers start their stopwatches some time after the runners have started because of the time taken for the sound of the pistol to reach them. Given that the speed of sound in air is around 340 $m\,s^{-1}$, calculate the size of this delay.

b. Is this a systematic error or a random error? Explain your answer.

c. The time for the winner is given by the time keeper as 12.72 s. The physics teacher is critical of this. Explain why and rewrite the time in a way that can be justified scientifically.

3. An electric kettle is used to bring water to the boil. The temperature of the water is measured with an electronic thermometer every 30 s. The results are shown in Table Q1.

a. A student has made a number of mistakes in recording the results. Suggest **two** corrections.

b. Plot a graph of temperature (y-axis) against time (x-axis).

Time	Temperature
0	10
30	35.3
60	54.7
90	72.4
120	87
150	95
180	100.2
210	100.2

Table Q1

c. Use the graph to calculate the greatest rate of increase of temperature.

d. Explain the shape of the graph.

4. A metal cube of side length 4.0 cm is manufactured to a tolerance of ±0.1 cm. Its volume will be:

A (64.0 ± 0.1) cm^3

B (64.0 ± 0.2) cm^3

C (64 ± 5) cm^3

D (64 ± 7.5) cm^3

5. The speed limit on British motorways is 70 mph. In SI units this would be written as:

A 31.1 $m\,s^{-1}$

B 1.87 $km\,min^{-1}$

C 43.8 $km\,h^{-1}$

D 43.8 $m\,s^{-1}$

6. Density is measured in kilograms per cubic metre. Water has a density of 1000 $kg\,m^{-3}$. What is the mass of 1 litre of water?

A 100 kg

B 10 kg

C 1 kg

D 100 g

7. Estimate the mass of a five-door family hatchback car. Which of these values is closest to the actual value?
   A 100 kg
   B 500 kg
   C 1000 kg
   D 5000 kg

8. Pressure is defined as the force on a certain area. Which of these would be the correct unit to measure pressure?
   A pound per square inch
   B kilogram per square metre
   C newton per cubic metre
   D newton per square metre

9. Estimate how many footballs you could fit into your (empty) classroom. Choose from:
   A 300 000
   B 30 000
   C 3000
   D 300

10. An experiment using polarised light requires a sugar solution of strength 100 g of sugar per litre of water. You are provided with a measuring cylinder of capacity 50 cm^3, marked in cm^3, and an electronic balance sensitive to 1 g. The maximum strength of your solution could be:
    A 100.3 g cm^{-3}
    B 100.2 g cm^{-3}
    C 102 g cm^{-3}
    D 103 g cm^{-3}

11. In an experiment a student measures the wavelength, λ, of different frequencies, f, of sound. The velocity of sound, v, is given by velocity = frequency × wavelength, $v = f \times \lambda$. To find a value for the velocity from the gradient of a graph, what should the student plot? Choose the correct row from Table Q2.

	y-axis	x-axis	Gradient
A	f	λ	v
B	λ	f	v
C	f	$\frac{1}{\lambda}$	v
D	$\frac{1}{\lambda}$	$\frac{1}{f}$	v

Table Q2

12. A student needs to measure the dimensions of a mobile phone as precisely as possible. Which of the rows in Table Q3 would be the most appropriate measuring devices?

	Length	Width	Thickness
A	Ruler	Vernier callipers	Micrometer
B	Ruler	Micrometer	Vernier callipers
C	Ruler	Vernier callipers	Micrometer
D	Micrometer	Micrometer	Vernier callipers

Table Q3

13. Two students are measuring the current through a circuit. Student A has a digital meter, which reads 0.1 A when the circuit is off. Student B has an analogue meter, which he views from an angle, leading to a parallax error. Which row in Table Q4 correctly describes the nature of these errors?

	Student A	Student B
A	Random	Random
B	Systematic	Random
C	Random	Systematic
D	Systematic	Systematic

Table Q4

14. Which row in Table Q5 correctly names the part of the micrometer screw gauge in Figure Q1 and correctly identifies its function?

	Part	Name	Function
A	6	Spindle	To clamp the specimen tightly
B	7	Ratchet	To slip, rather than over-tighten and deform the specimen
C	4	Ratchet	To lock the jaws
D	1	Sleeve	To measure the specimen

Table Q5

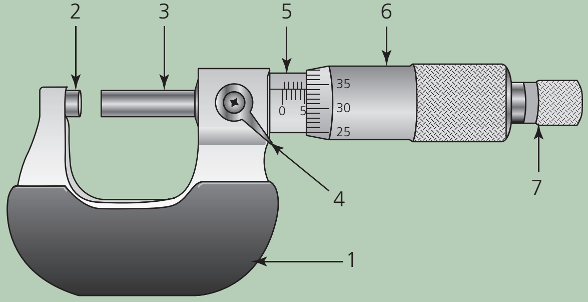


Figure Q1

15. The micrometer in Figure Q1 is reading:

A 5.34 mm

B 5.534 mm

C 5.84 cm

D 5.34 cm

16. A micrometer like that in Figure Q1 has:

A A range of 25 cm and a resolution of 0.1 mm

B A range of 25 mm and a resolution of 0.01 mm

C A range of 25 mm and a resolution of 0.1 mm

D A range of 2.5 mm and a resolution of 0.01 mm

17. Vernier callipers are to be used to measure a short pipe.

Which of the following statements is **false**?

A Vernier callipers can be used to measure the internal and external diameter of the pipe.

B Vernier callipers have better resolution than a micrometer.

C Vernier callipers have a larger range than a micrometer.

D Vernier callipers can measure to the nearest 0.1 mm.

18. Look at the calliper scales in Figure Q2.

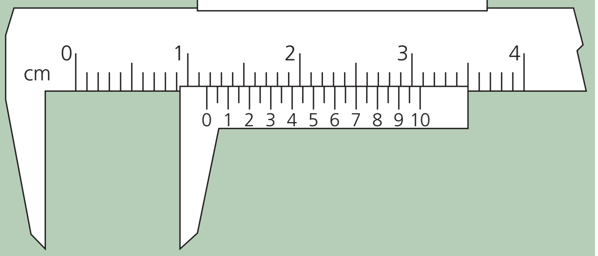


Figure Q2

What is the reading on the callipers?

A 1.07 cm

B 1.15 cm

C 7.25 cm

D 1.17 cm