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Cambridge IGCSE®

Biology

Maths Skills Workbook

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Contents

Introduction

This workbook has been written to help you to improve your skills in the mathematical processes that you need in your Cambridge IGCSE Biology course. The exercises will guide you and give you practice in:

- representing values
- working with data
- drawing graphs and charts
- interpreting data
- doing calculations
- working with shape.

Each chapter focuses on several maths skills that you need to master to be successful in your biology course. It explains why you need these skills. Then, for each skill, it presents a step-by-step worked example of a question that involves the skill. This is followed by practice questions for you to try. These are not like exam questions. They are designed to develop your skills and understanding; they get increasingly challenging. Tips are often given alongside to guide you. Spaces, lines or graph grids are provided for your answers.

In biology, there are lots of contexts where maths is used. You will be calculating magnification and using scale when working with microscopes. Probability and ratio are used to interpret the results from genetic crosses. An important skill is analysing data in the form of tables, graphs and charts. This could be data that you, or other scientists, have collected during an investigation.

Some of the maths concepts and skills are only needed if you are following the Extended syllabus (Core plus Supplement). The headings of these sections are marked 'Supplement'. In other areas just one or two of the practice questions may be based on Supplement syllabus content, and these are also clearly marked.

There are further questions at the end of each chapter that you can try to give you more confidence in using the skills practised in the chapter. At the end of the book there are additional questions that may require any of the maths skills from all of the chapters.

Chapter 3:
Drawing graphs and charts

Why do you need to be able to draw graphs and charts in biology?

- Biologists use graphs and charts to display data that they have collected. This makes it easier to compare data and see patterns.
- There are different types of graph and chart used in biology; these include bar charts, pie charts, histograms, line graphs and scatter graphs.
- The type of graph or chart chosen depends on the type of data.

Maths Focus 1: Drawing bar charts

Bar charts

LINK
See Chapter 2, Maths focus 1 'Naming types of data', for more information on these types of data.

Bar charts are used to show data that can be sorted into different categories. This might be categorical or discrete data.

A bar chart can be used to compare the amount of fat in different foods, see Figure 3.1.

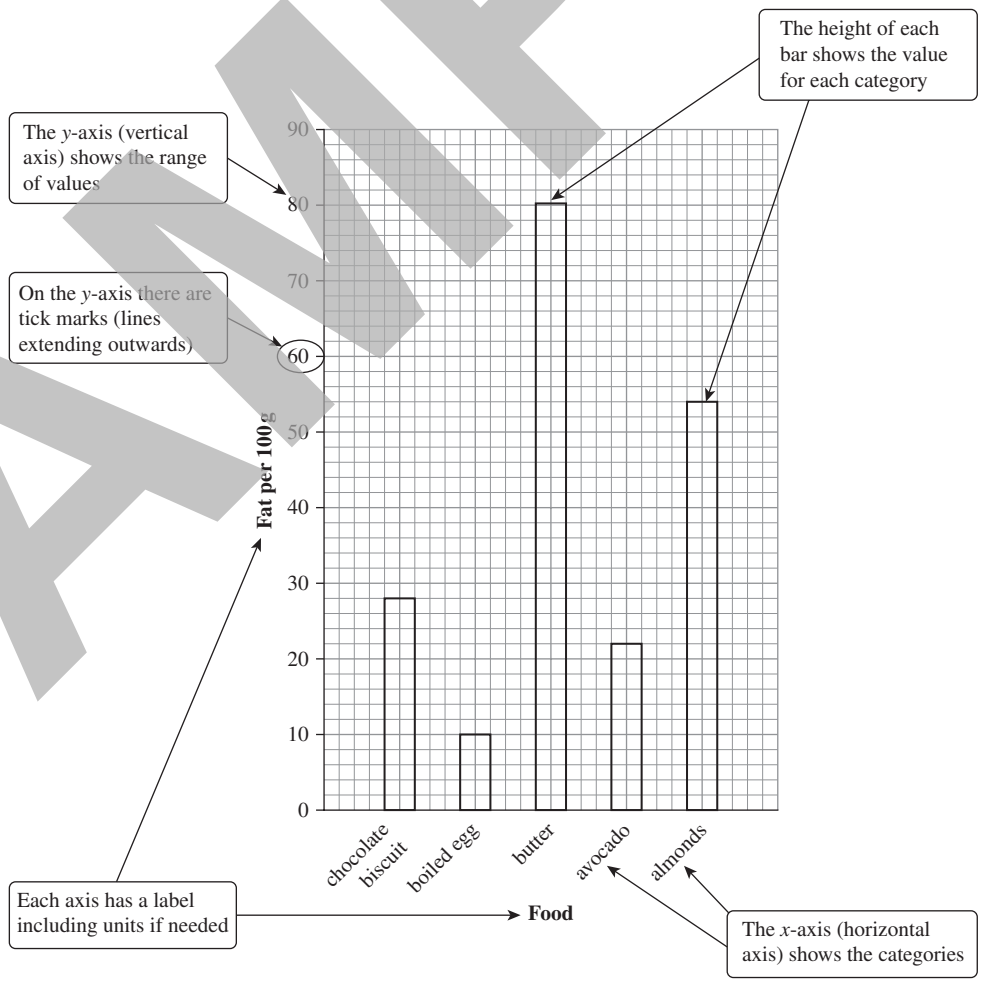


Figure 3.1 A bar chart to show the amount of fat per 100 g of different foods

What maths skills do you need to draw a bar chart?

1 Choosing a suitable scale for the *y*-axis	• Choose the scale so all the data can be included • Aim to use as much of the graph paper as you can • Avoid scales that make the values hard to read
2 Drawing the bars	• Show each category by one bar • Make all the bars the same width and separate them with a gap • Draw the bars as accurately as you can to the correct height

Maths skill practice

How does drawing bar charts relate to discontinuous variation?

In biology you might collect data on variation in a group of people, animals or plants. Some of this data will be discontinuous, which means it can be sorted into categories. For example, a person's blood group is either A, B, AB or O. This is shown in Table 3.1.

Blood group	Number of people
A	24
B	6
AB	2
O	28

Table 3.1 Number of people with the different blood groups

Drawing a chart will show more clearly how many people have each blood group so you can compare them. The data is *categorical* so can be displayed using a bar chart.

Maths skill 1: Choosing a suitable scale for the y-axis

In the bar chart showing blood groups, the *y*-axis is going to display the *number of people*.

The lowest number of people is 2 and the highest is 28.

It is always best to start the *y*-axis at 0 (unless all the numbers are very large). So, for this bar chart the *y*-axis will start at 0 and go up to at least 28.

Graph paper is normally divided up into large squares, see Figure 3.2. Each square contains many smaller squares, normally: $10 \times 10 = 100$

The side of each large square on the graph paper should have a value of 1, 2 or 5 multiplied by a power of 10.

WATCH OUT

In some countries graph paper is also called *millimetre* paper, because each little square is 1 mm × 1 mm.

TIP

The scale you choose depends on how big the numbers are that you need to show.

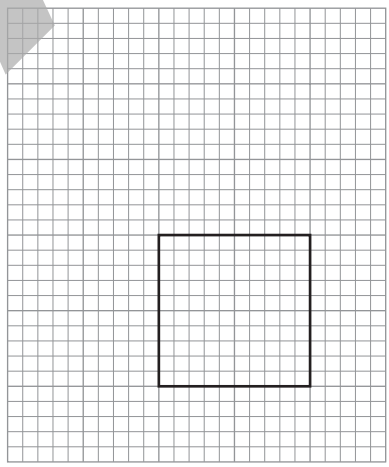


Figure 3.2 One large square on this graph paper

For example, you could choose:

	0.1	1	10	100, etc.
or	0.2	2	20	200, etc.
or	0.5	5	50	500, etc.

WORKED EXAMPLE 1

For the data on blood groups in Table 3.1, which of the scales shown in Figure 3.3 would you choose? Explain your choice.

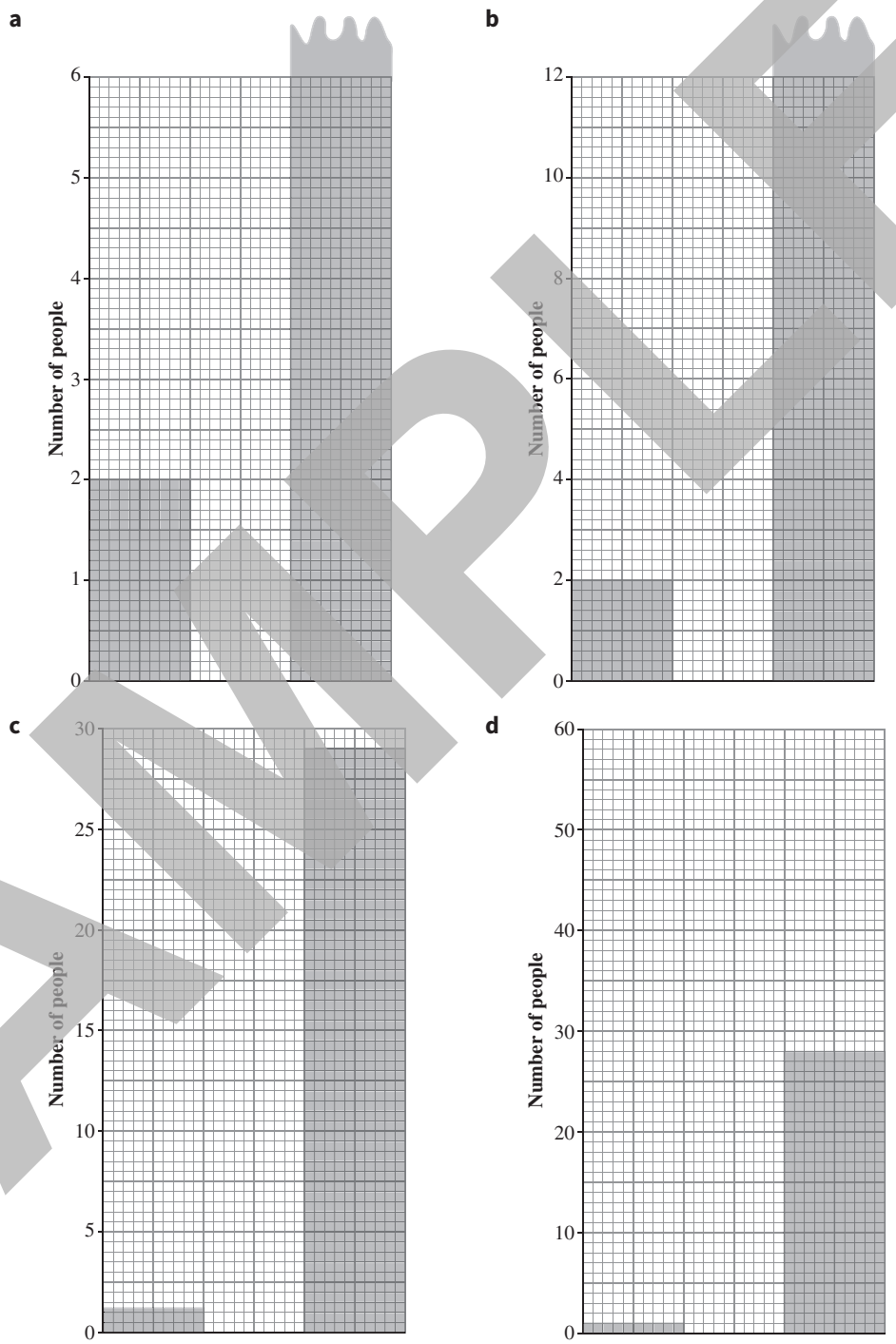


Figure 3.3 Different scales for drawing the *y*-axis

The scale in **C** would be the best choice.

This is because in A and B the scales mean that the bar for 28 people will not fit on the paper.

The scale in D results in the bar for two people being too small to read easily.

TIP

The **axis** with the scale you have chosen should take up over half of the space you have given, whether this is a whole sheet of graph paper or the graph paper drawn on an exam paper.

WATCH OUT

Make sure you leave enough space to write the title of the *y*-axis next to it.

Practice question 1

Some people can roll their tongue and others cannot.

The number of students who can or can't roll their tongue in a class was counted.

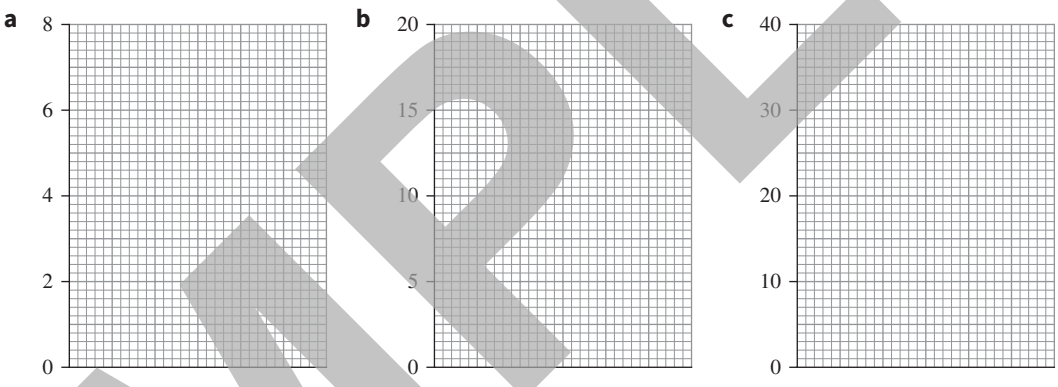
The data are shown in Table 3.2.

Tongue roller	Number of students
yes	18
no	12

Table 3.2 Results for a survey on tongue rolling

Which *y*-axis scale (A–C) is the best choice to display this data?

Draw a circle around the correct letter.



Practice question 2

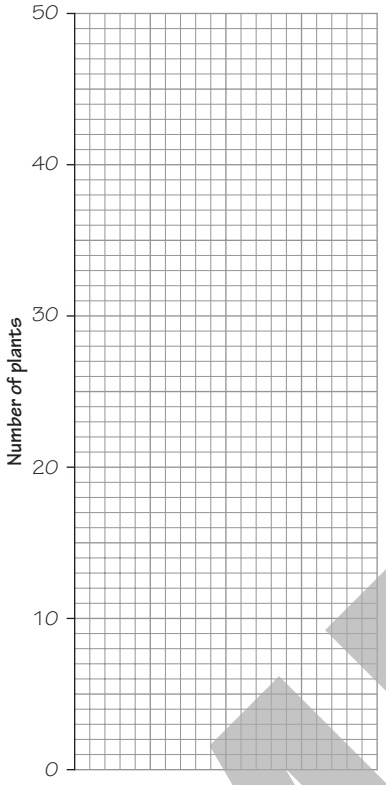
A student collected data on the colour of flowers on different pea plants.

His data are shown Table 3.3.

Colour of flowers	Number of plants
white	82
yellow	26
red	14

Table 3.3 Results for a survey on the colour of flowers

He starts to write the scale on the *y*-axis on the graph paper as shown below.



a Explain what he is doing wrong and why.

.....

.....

.....

b Suggest how he should draw the scale.

.....

Practice question 3

Shoes come in different sizes. You can only be one shoe size.

A student collected data on the shoe size of the girls in her class.

Table 3.4 shows the data she collected.

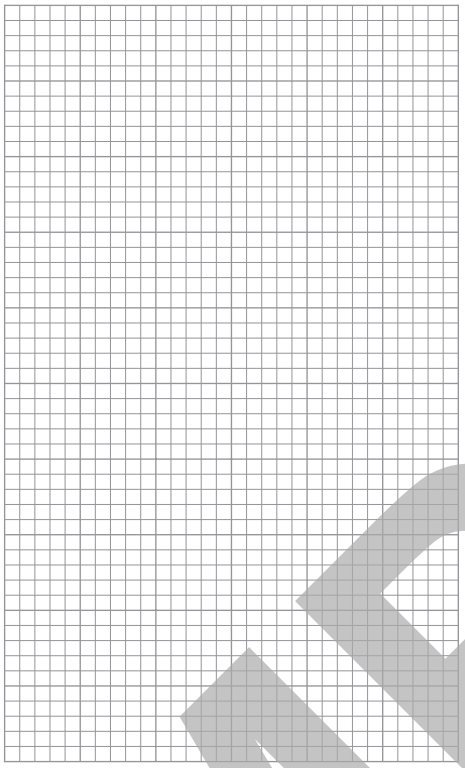
Shoe size	Number of girls
35	0
36	3
37	10
38	6
39	6
40	4
41	1
42	0

Table 3.4 Results for a survey on shoe size

TIP
The categories can be numbers. This is called *discrete* data.

TIP
The *y*-axis is drawn vertically using a ruler.

On the following graph paper, draw a suitable *y*-axis which can be used to show this data as a bar chart.



Maths skill 2: Drawing the bars

Draw bars to extend from the *x*-axis.

Each bar represents one category.

The height of the bar shows the value for each category.

WORKED EXAMPLE 2

If we return to the blood group data from Table 3.1, we can see the steps needed to draw the bars.

Blood group	Number of people
A	24
B	6
AB	2
O	28

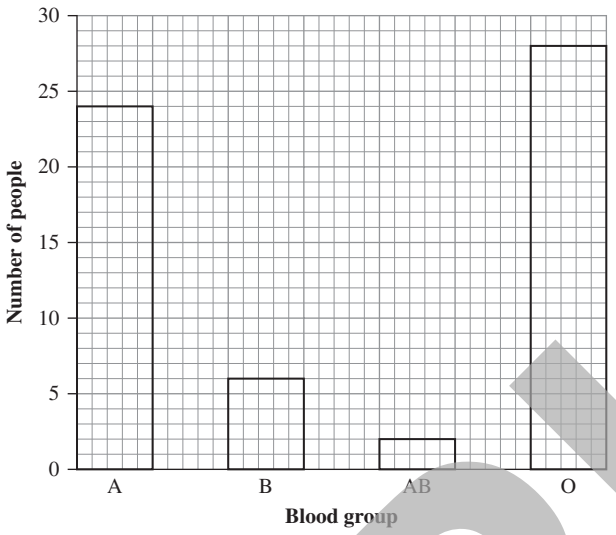


Figure 3.4 A bar chart to show the number of people with different blood groups

Step 1: Draw a line for the *x*-axis.

KEY QUESTIONS TO ASK YOURSELF:

1 How many bars will you need to draw?

There are four blood groups, so there will be four bars.

2 How wide will each bar be?

Work out how you will fit the four bars on your grid.

3 How much space will you leave between each bar?

Allow the same amount of space between each.

TIP

Make sure all the bars will fit onto the graph paper.

The bars all need to be the same width.

Step 2: Draw the bars in order of the rows in the table. So, for this chart the first bar will show the number of people with blood group A. Use a ruler to draw the first bar next to the *y*-axis.

Use the scale to work out where the top of the bar should be. Using this scale, two *small* squares represent 1 person. There is no need to colour the bars in.

Step 3: Underneath the bar, write the name of the category.

Step 4: Leave a gap and draw the next bar. The size of the gap is not important, as long as the bars are not touching.

Step 5: Underneath the *x*-axis, write the label (copy this from the table). For this chart, it is 'Blood group'.

Practice question 4

A student collected data on the number of boys in each year in his school.

His data are shown in the table.

Year	Number of boys
7	120
8	89
9	101
10	117
11	95

He draws a bar chart.

a State the title of the *x*-axis. ……..

b State the name of the first bar he will draw. ……..

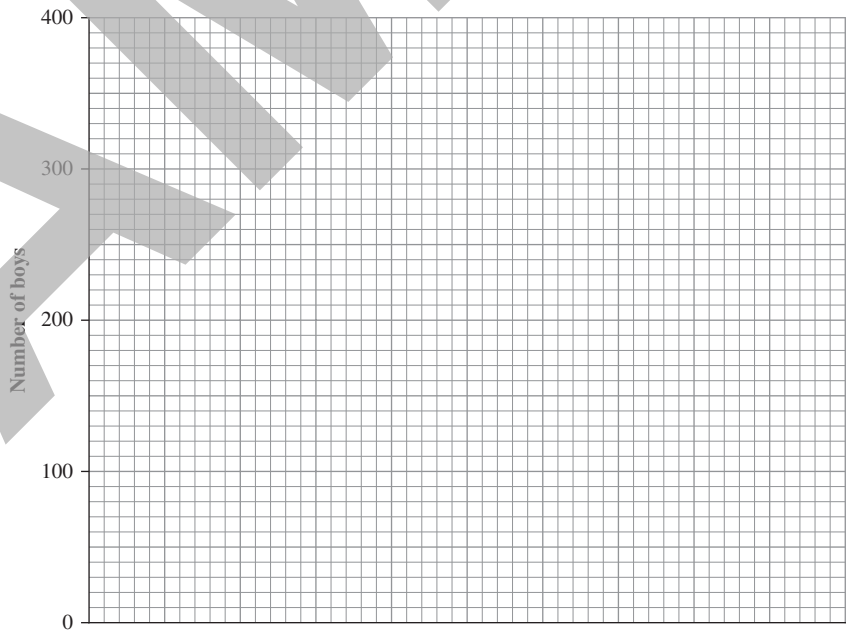
Practice question 5

The student then collected data on how many boys in the school were left or right handed.

His data are shown in the table.

Handedness	Number of boys
Right	354
Left	168

Complete the bar chart to show the data.



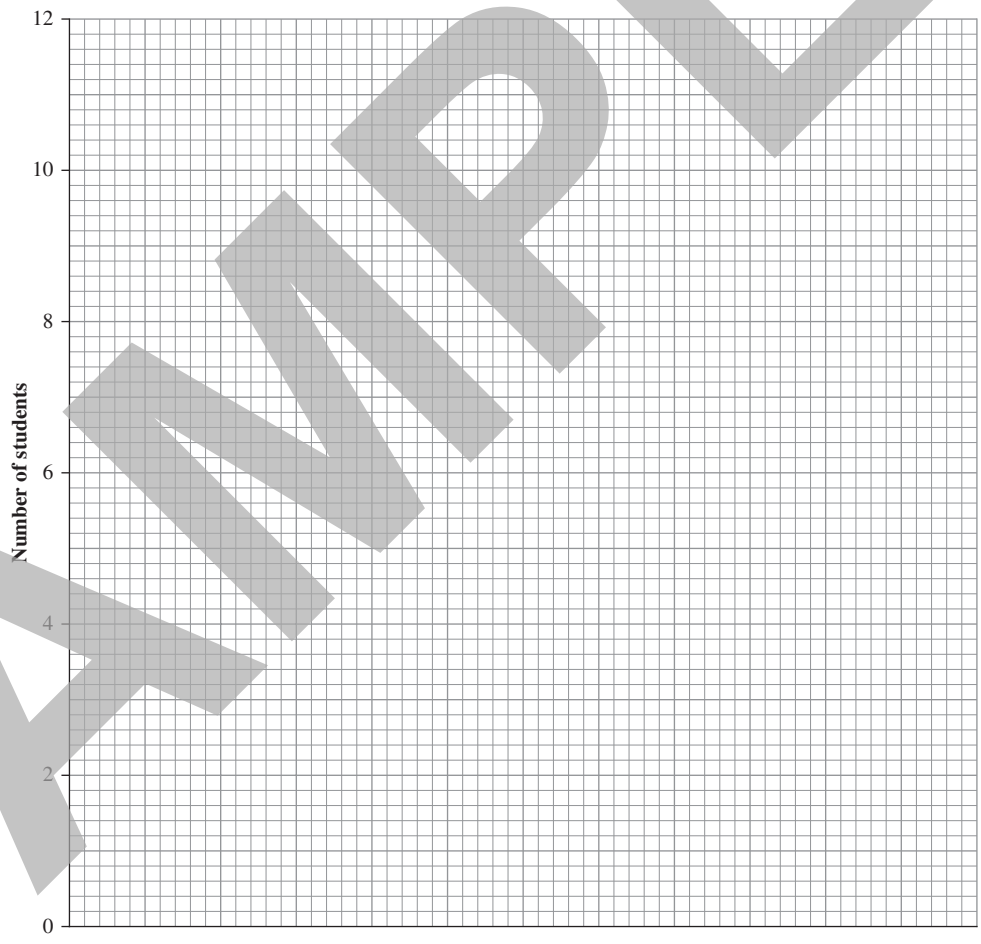
Practice question 6

The student collected data on how many brothers or sisters (siblings) the students in his class had.

His data is shown in the table.

Number of siblings	Number of students
0	2
1	12
2	11
3	4
4+	1

Draw a bar chart to show the data below.



Maths focus 2: Drawing histograms

A **histogram** is used to display the distribution, or spread, of continuous data.

The data in Table 3.5 shows the masses of some bananas.

You could draw a bar chart to show the mass of each banana but that would contain a lot of bars. Because mass is a continuous variable, you can group them together into groups called **classes**. This is displayed in a **frequency table**, as in Table 3.5.

The first column shows the *classes*.

The size of the class is called the class interval. In this case it is 10.

The second column shows the *frequency*.

This is how many bananas are in each class.

Mass/g	Frequency
110–119	3
120–129	6
130–139	5
140–149	2

Table 3.5 Frequency table of masses of bananas

This data can be used to draw a histogram, which shows the spread of the data as in Figure 3.5.

WATCH OUT

There are differences between the meaning of 'histogram' in mathematics and science. In mathematics the widths of each column are different to each other, because their *area is equal to the value*. In science the bars are the same height, so the area is *not* equal to the value.

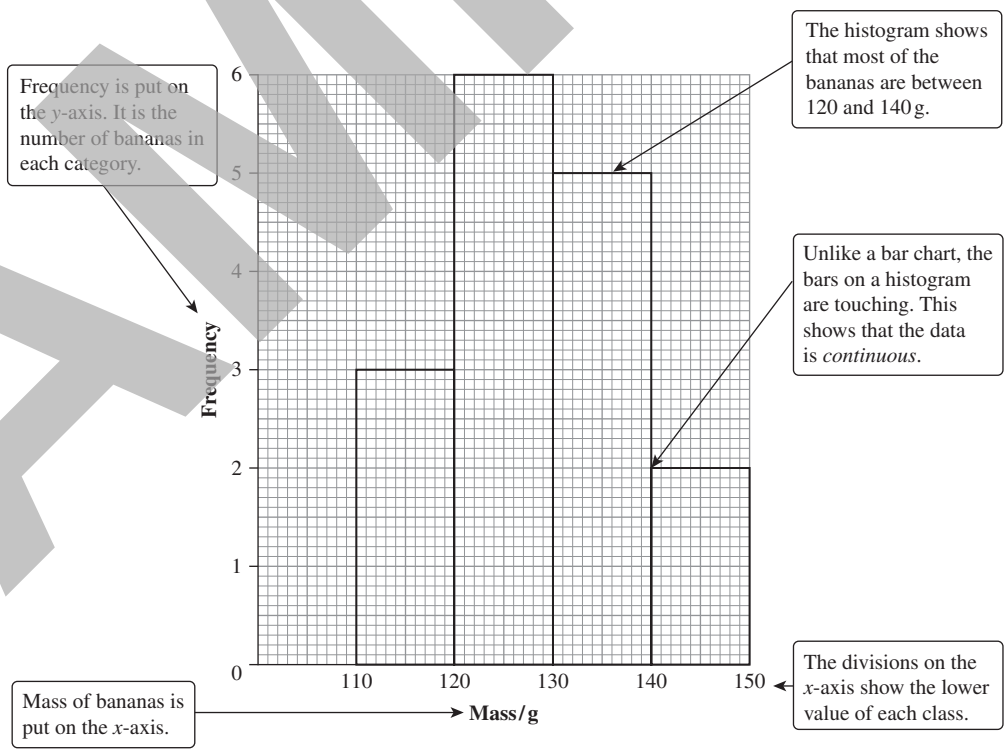


Figure 3.5 A histogram to show the distribution in mass of some bananas

What maths skills do you need to draw a histogram?

1 Putting the data into classes	• Choose the class interval so there are neither too few nor too many classes • The frequency of each class is worked out
2 Drawing the histogram	• The classes are put on the x-axis • The bars are drawn to show the frequency of each class • The bars must be touching

Maths skill practice

How does drawing histograms relate to continuous variation?

When you study variation, some of the data you collect will be continuous. Examples include the height of plants, the hand-span of people or the mass of fruit.

Maths skill 1: Putting the data into classes

TIP

Your classes should not overlap. For example, you cannot choose classes of 140–150 cm and then 150–160 cm, because then it is not clear in which class a height of 150 cm would be placed.

WORKED EXAMPLE 3

The data below shows the height of a group of 15–16 year olds.

Height/cm	154, 156, 164, 151, 142, 168, 165, 170, 156, 151, 145, 142, 158, 171, 149, 165, 169, 157

Drawing a histogram will show more clearly the distribution of height in the class. This will show if more people are shorter or taller, and the most common height. This pattern can then be compared to another class, or even the whole country.

Step 1: Write the data out in order:

Height/cm	142, 142, 145, 149, 151, 151, 154, 156, 156, 157, 158, 164, 165, 165, 168, 169, 170, 171

Step 2: Now you can choose your class intervals. You should choose a size that gives you not too few or too many classes. A total of 4–6 classes is a good number.

Height/cm	142, 142, 145, 149,	151, 151, 154, 156, 156, 157, 158,	164, 165, 165, 168, 169,	170, 171
Classes	140–149 cm	150–159 cm	160–169 cm	170–179 cm
Freq.	4	7	5	2

Step 3: Finally, you can work out the frequency in each class. This is how many heights fall into each class. For example, there are 4 in the 140–149 cm class.

Practice question 7

The data shows the length of the middle finger of a group of women.

Length of middle finger/cm	7.7	6.8	6.5	7.9	8.1	7.5	7.2	6.6	7.8	6.4	7.9	8.0	7.5	7.9	8.2

Complete this frequency table.

Length of middle finger/cm	Frequency
6.0–6.4	
6.5–6.9	
7.0–7.4	
7.5–7.9	
8.0 – 8.5	

Practice question 8

The data shows the mass of a collection of tortoises living in a zoo.

The zoo keeper wants to display the data as a histogram.

Mass of tortoise/g	125	101	123	130	142	100	155	158	154	146	132	129

a The zoo keeper starts to draw a frequency table.

Complete the classes in the first column.

Mass of tortoise/g	Frequency
100–114	

b Suggest why the zoo keeper chose this class interval.

.....

Practice question 9

A student measured the length of the leaves on a bamboo shoot.

The table shows her data, in order of length.

Length of leaf/mm	50	51	53	57	59	63	63	64	66	68	70	71	72	72	73	73

Choose suitable class intervals and draw a frequency table to display the data.

Maths skill 2: Drawing the histogram

WORKED EXAMPLE 4

Let's return to the data we looked at in Maths skill 1, Worked example 3 about the height of a group of 15–16 year olds.

Here is the frequency table for the data.

Height/cm	Frequency
140–149	4
150–159	7
160–169	5
170–179	2

Step 1: Draw the *y*-axis.

Frequency is plotted on the *y*-axis, so look at the highest and lowest frequency in the table.

In this example, the highest frequency is 7. Each large square has the value of one.

Make sure you label the y-axis 'frequency'.

WATCH OUT

You should start the *y*-axis at 0.

TIP

The bars don't have to be one large square wide; you can make them narrower or wider. Try to fill the graph paper you have been given.

Step 2: Draw the x-axis.

The divisions on the x-axis show the class intervals.

The first number will be the smallest value of your first class.

In this example it is 140 cm. You do not have to start this axis at 0.

The lowest value of the next class is 150, so this is the value you write in the next large square. Each large square in this histogram has the value of 10.

Label the x-axis with the variable and unit.

Step 3: Draw the bars.

The height of each bar represents the frequency of that class, see Figure 3.6.

Unlike a bar chart, the bars need to be touching.

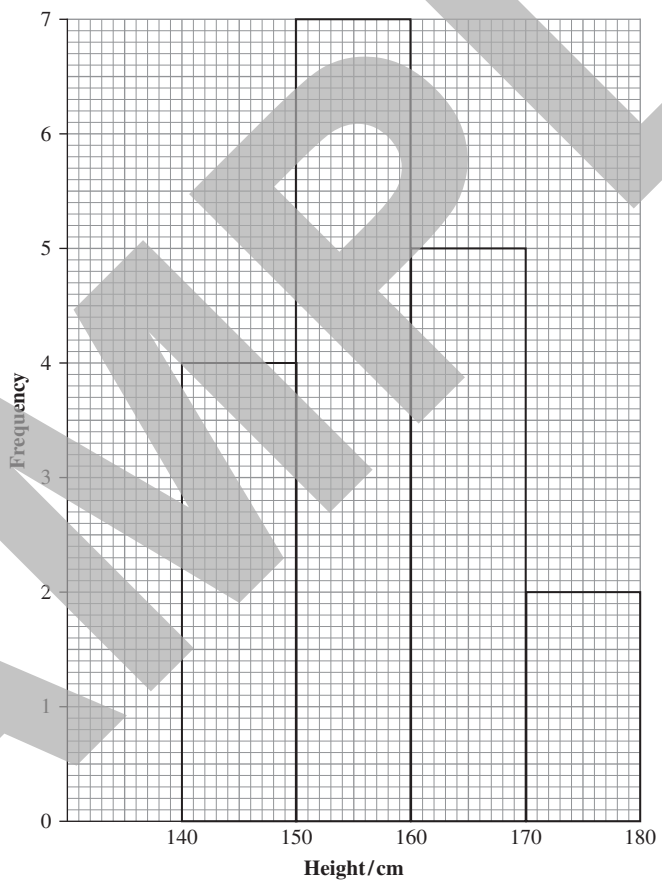


Figure 3.6 A completed histogram to show the heights of a class of students

WATCH OUT

The data in the table uses decimal numbers, but the method used is exactly the same as with whole numbers.

Practice question 10

A student collected data on the students in her class.

She started with hand span. The data she collected is shown in the frequency table.

Hand span/cm	Frequency
15.0–16.9	2
17.0–18.9	5
19.0–20.9	8
21. 0–22.9	5
23.0–24.9	3

Complete the histogram on the axes below.

Practice question 11

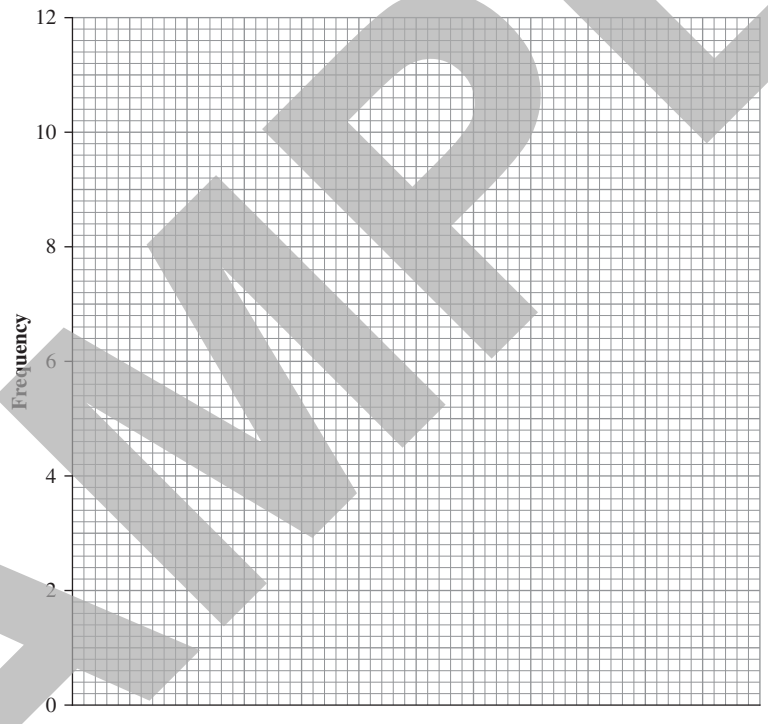
The student then measured the length of the students' right feet.

The data she collected is shown in the frequency table.

Foot length/mm	Frequency
200–219	1
220–229	4
230–239	12
240–249	6
250–259	1

a On the graph below, draw the scale and title for the *x*-axis.

b Draw the bars to complete the histogram.



Practice question 12

Finally, she measured the resting heart rate of the students in her class.

The data are shown in the table.

Resting heart rate/beats per minute	Frequency
50–59	1
60–69	9
70–79	8
80–89	9
90–99	4
100–109	3

Draw a histogram to show the data on the graph paper below.

LINK
See Chapter 2, Maths focus 3 'Recording and processing data' for more information.

Maths focus 3: Drawing line graphs

Line graphs are very common in biology. They are used to show the relationship between two continuous variables: the independent and the dependent variable.

For example, a line graph can be used to show how the mass of a foetus changes over the weeks it is in the womb (see Figure 3.7).

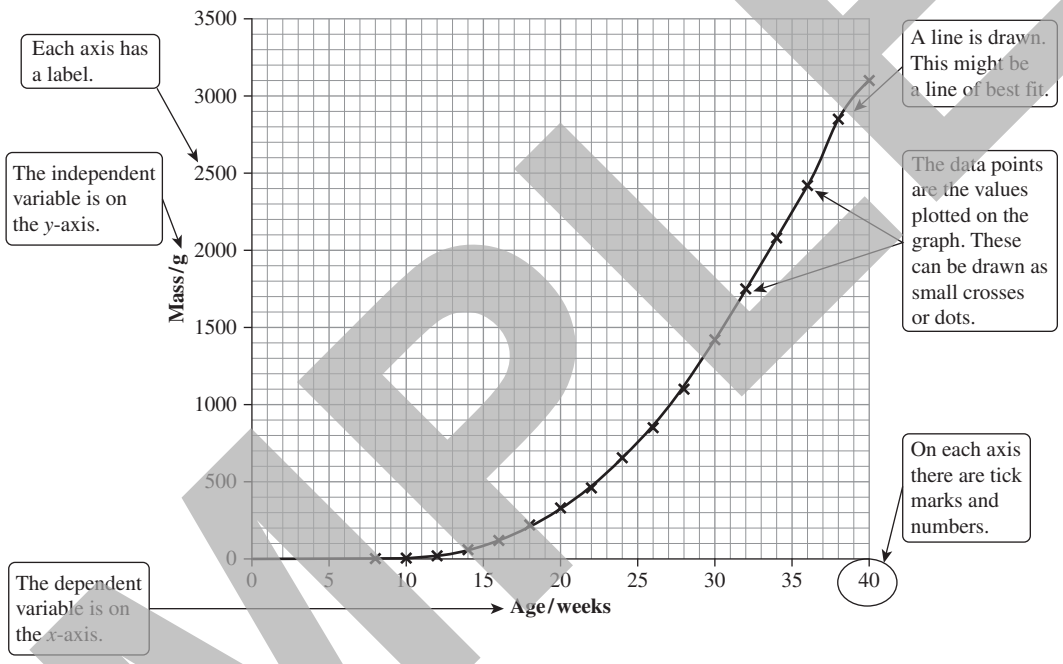


Figure 3.7 A graph to show the change in mass of a foetus

This line graph shows how the mass (in grams) of a foetus changes with its age (in weeks).

The mass of the foetus is the dependent variable. The age of the foetus is the independent variable.

Graphs in biology can show how something varies over time. Here, time is the independent variable and so is plotted on the *x*-axis.

What maths skills do you need to draw a line graph?

1 Drawing the axes	• Deciding which variable goes on which axis • Choosing the range of each axis • Choosing an appropriate scale
2 Plotting the data points	• How to accurately plot each data point
3 Drawing the line or curve of best fit	• Knowing how to draw a line or curve of best fit

LINK
More about interpreting line graphs is covered in Chapter 4, 'Interpreting data'

LINK
See Maths focus 1, Maths skill 1, 'Choosing a suitable scale for the *y*-axis' to remind yourself about this.

TIP
You should start each axis at the **origin** (0, 0).

WATCH OUT
For some graphs an axis might contain negative numbers. Figure 3.9 shows an example.

TIP
Don't forget to label the axes including units. You can use the headings from the table.

Maths skill practice

How does drawing line graphs relate to photosynthesis?

When you study photosynthesis you will carry out many investigations and gather data.

You will investigate how different variables, such as light intensity and temperature, affect the **rate** of photosynthesis (i.e. how quickly it takes place).

Using the data to draw line graphs will help you to work out relationships between the variables and also calculate how they affect the rate of photosynthesis.

Maths skill 1: Drawing the axes

WORKED EXAMPLE 4

A student investigated the relationship between light intensity and volume of oxygen produced by the pondweed *Elodea* during photosynthesis.

Figure 3.8 shows the equipment he used.

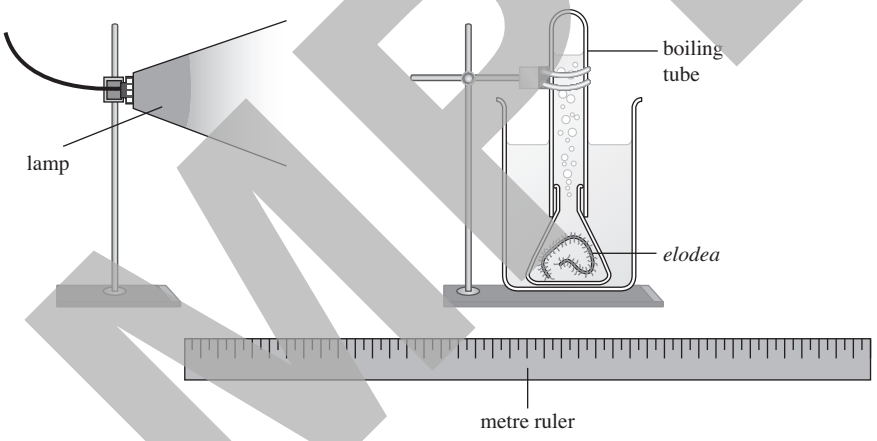


Figure 3.8 The equipment used to investigate how light intensity affects the rate of photosynthesis

Table 3.6 shows the results.

Distance between lamp and plant/cm	Number of bubbles produced in 1 minute
10	56
20	28
30	24
60	18
80	10
100	6

Table 3.6 Results from an investigation into how light intensity affects the rate of photosynthesis

Draw the axes for a graph of this data.

WATCH OUT

In the table, the values for the dependent variable do not increase by equal amounts: 10, 20, 30, 60, 80, 100. You must *not* plot these values on the *x*-axis, but instead make sure each large square has the same value, such as 0, 20, 40, 60, 80, 100.

Sometimes the independent variable in a table might not be in numerical order. For example, the student may have chosen to use a distance of 100 cm, not 10 cm first. Even if this is the case, you must always draw the axes so they start at 0 and then increase as you go along.

Step 1: First, you need to decide which is the dependent variable and which is the independent variable.

In this example, the distance the lamp was from the plant was the variable being changed by the students so this is the *independent* variable. It goes on the *x*-axis.

The number of bubbles produced in 1 minute changed as a result of moving the lamp, so this was the *dependent* variable. This goes on the *y*-axis.

Step 2: Next, you will need to work out a suitable scale for each axis. This is the same skill you learnt about previously when drawing bar charts. The only difference is, with a line graph, you will also need to choose a scale for the *x*-axis.

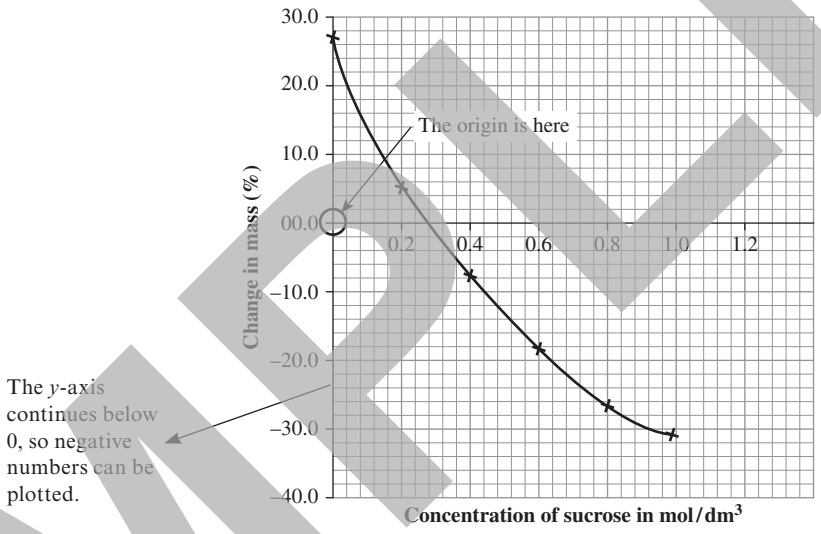


Figure 3.9 Change in mass of potato at different sucrose concentrations

Figure 3.10 shows axes suitable for plotting a line graph of this data.

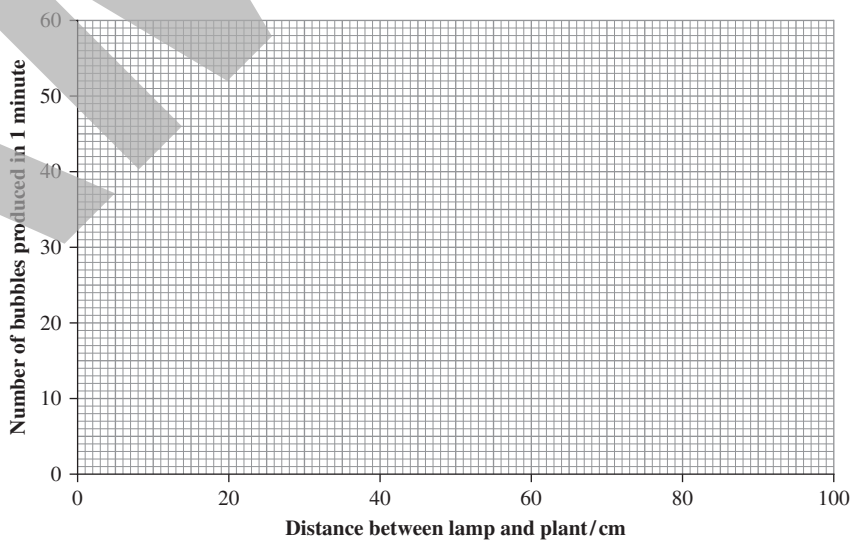


Figure 3.10 The *x* and *y*-axes for an investigation into how light intensity affects the rate of photosynthesis

Practice question 13

A student investigated how the amount of carbon dioxide affected the rate of photosynthesis.

He changed the concentration in the water surrounding a piece of *Elodea* and kept the light intensity the same. He measured the volume of oxygen produced in one minute.

a What independent variable did he use? Circle the letter of your choice.

A Light intensity

B Length of the *Elodea*

C Volume of oxygen produced in 1 minute

D Concentration of carbon dioxide

b After collecting his results, he decided to draw a line graph.

State what variable he should plot on the:

i *y*-axis .....

ii *x*-axis .....

Practice question 14

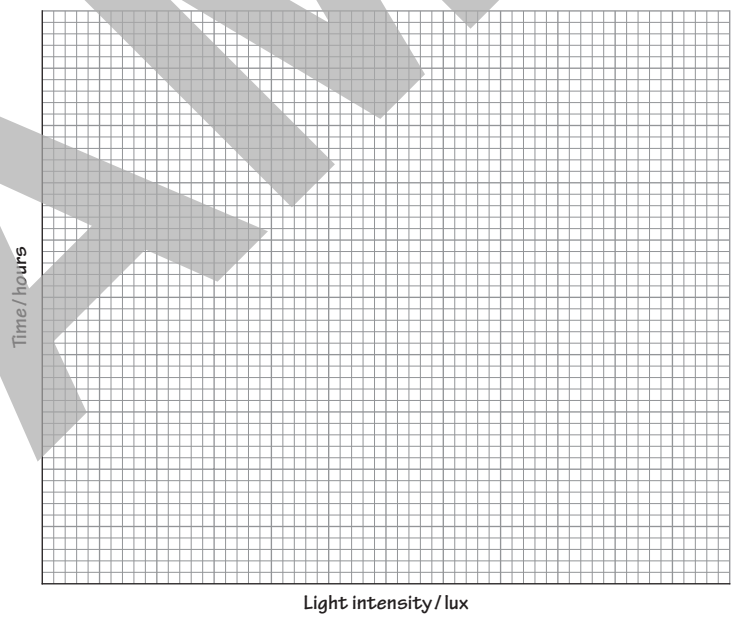
The light intensity (in lux) of a point in a forest was measured over a period of 24 hours.

The following sketch shows the axes labels for a line graph of the results.

Describe the mistake made in the labels.

.....

.....



Practice question 15

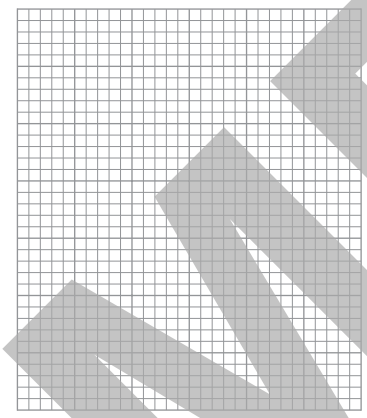
A student investigates the effect of temperature on the rate of photosynthesis.

The table shows his results.

Temperature/°C	Number of oxygen bubbles produced in one minute
10	16
20	25
30	33
40	43
50	0

Complete the axes below. You should:

- Decide which variable to plot on which axis.
- Choose a suitable scale for both axes and write numbers and tick marks on them.
- Write labels for each axis.



TIP

Each data point has an *x*-coordinate and a *y*-coordinate. The coordinates (*x*, *y*) show you the position of the data point on the axes.

Maths skill 2: Plotting the data points

WORKED EXAMPLE 5

Return to the data in Worked example 4; the first data point is in the first row of the table and is (10, 56). See Figure 3.11.

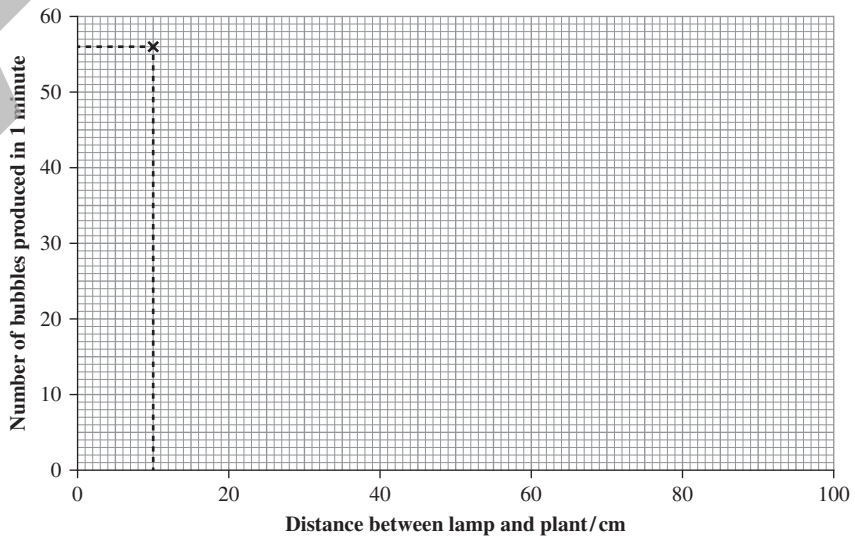


Figure 3.11 How to plot data points

TIP

On this graph paper, each large square contains 10 small ones. To work out what each small square represents, divide the value of the large square by 10. So, in this example the large squares on the *x*-axis have a value of 20 cm, so the small squares each have a value of $\frac{20}{10} = 2$ cm.

To plot this data point, find where 10 is on the x-axis and then travel up this line until you reach 56 on the y-axis. Draw a cross or a small dot with a circle drawn around it so the middle of the cross or dot is where these lines meet.

Carry on until you have plotted all the data points.

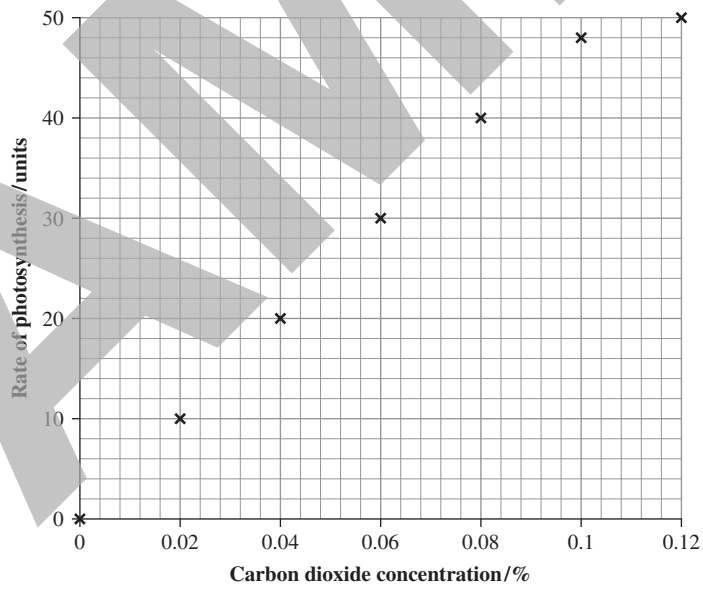
Use a ruler to guide you along the lines if you find it difficult.

Practice question 16

A student found the following results table on the internet.

Carbon dioxide concentration/%	Rate of photosynthesis/units
0	0
0.02	20
0.04	34
0.06	40
0.08	45
0.1	48
0.12	50

She plotted the data on a line graph.

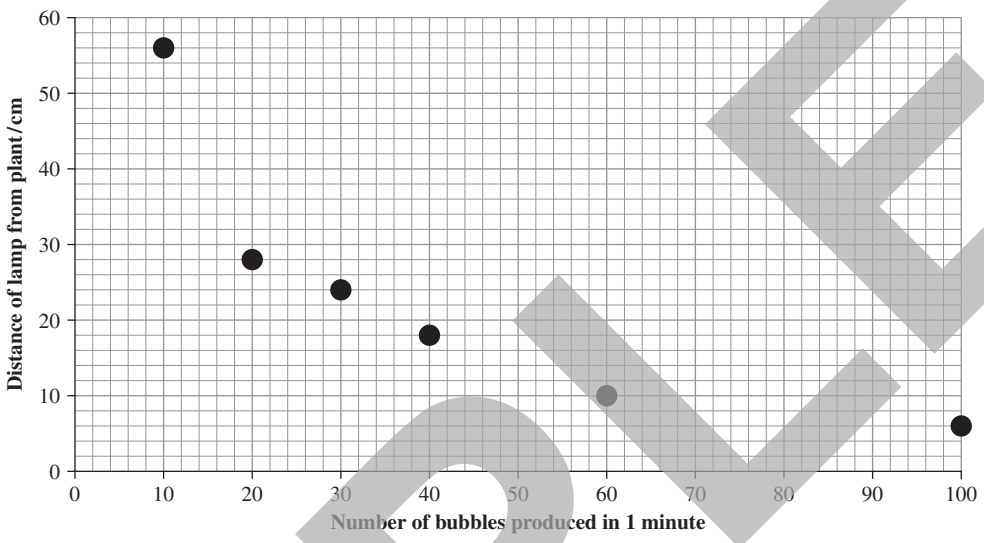


Circle any data points she has plotted incorrectly.

Practice question 17

A student plotted his results from an investigation on photosynthesis.

His graph is shown below.



a Describe what he has done wrong.

.....

b Explain what he should do to rectify his mistake and why it is important he does.

.....

.....

.....

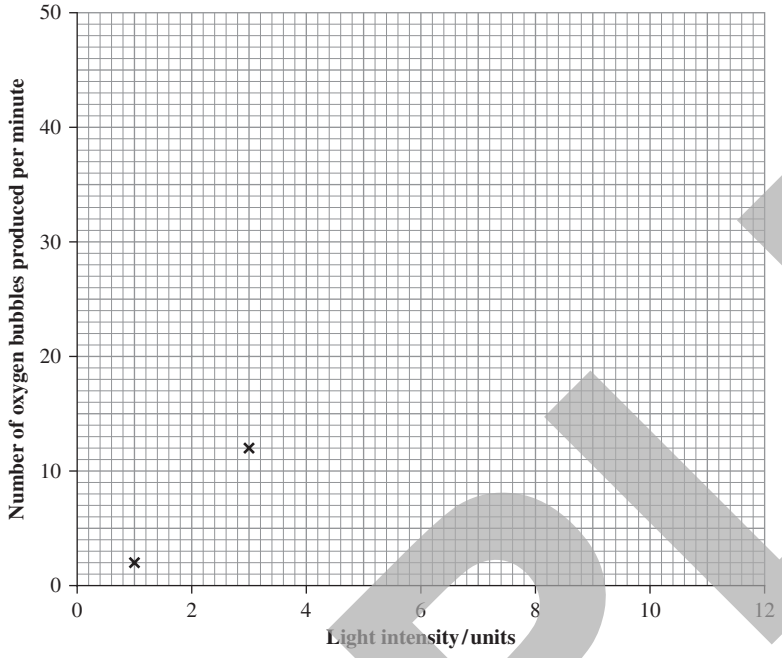
.....

Practice question 18

The table shows some results for an investigation on photosynthesis.

Light intensity/units	Number of oxygen bubbles produced per minute
1	2
3	12
5	24
8	38
10	45
12	45

Complete the following graph by drawing the remaining data points.



Maths skill 3: Drawing a line or curve of best fit

1 Joining the points

In biology the data you collect might be taken at intervals over time. For example, pollution levels once a month, or population size once every 5 years.

The data points should be joined with straight lines. The line could be straight (see Figure 3.12a), curved(see Figure 3.12b) or go up and down in a zig-zag fashion(see Figure 3.12c).

WATCH OUT

Only draw the line going through (0, 0) if there is a data point plotted there. Otherwise, start the line at the first data point and finish at the final one.

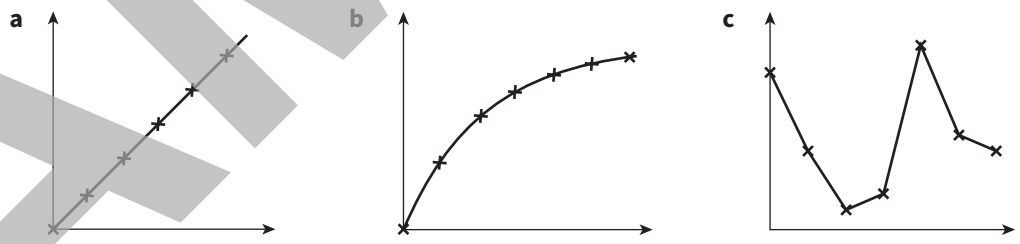


Figure 3.12 Graph **a** is a straight line graph, graph **b** is curved, graph **c** goes up and down

2 Drawing a best-fit line

Most of the line graphs you draw in biology are using results from an investigation where one variable affects another. In this case the data points will not be accurate because of errors, so you will draw a **best-fit line** to show the trend of the data.

WATCH OUT

There may be one or more *anomalous* (wrong-looking) data points in the results. Draw a circle around them. They should be ignored when drawing the best-fit line.

Figure 3.14 An anomalous result

TIP

A best-fit line is drawn so it passes close to all the points. Some of the points are on the line, some above and some below.

WORKED EXAMPLE 6

Follow these steps:

Step 1: Place a transparent ruler along the data points. This will allow you to see all the points so you can judge where to draw the line.

Step 2: Decide if the line should go through the origin (0, 0).

Step 3: Move the ruler so there are roughly the same number of points, evenly spread, above and below the line.

Step 4: Use a sharp pencil to draw the line, see Figure 3.13.

The line of best could be a curve. The same rules apply – try to draw a line that passes by most of the points.

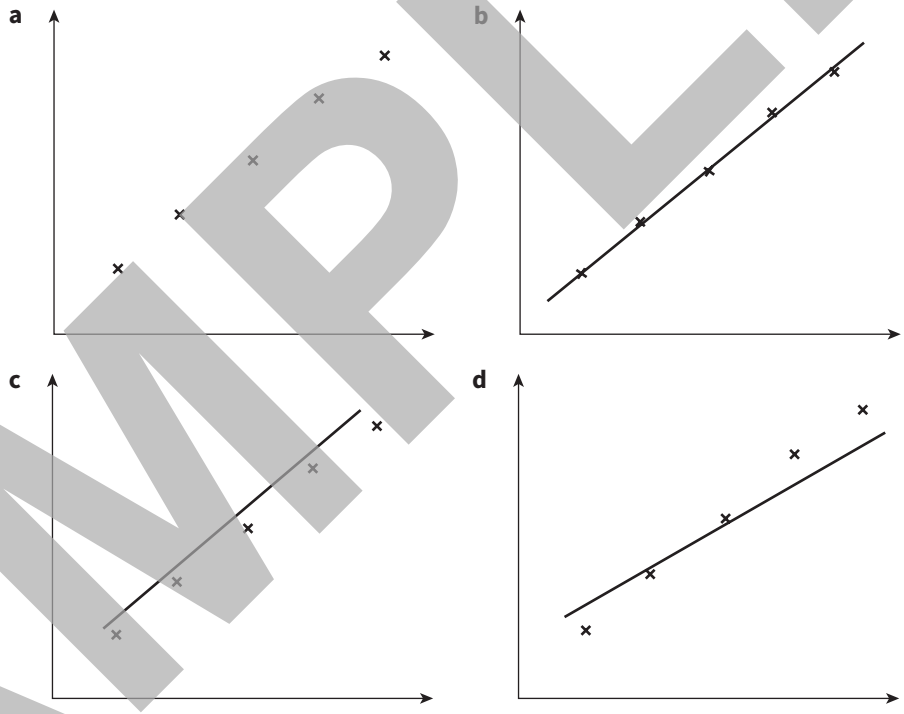
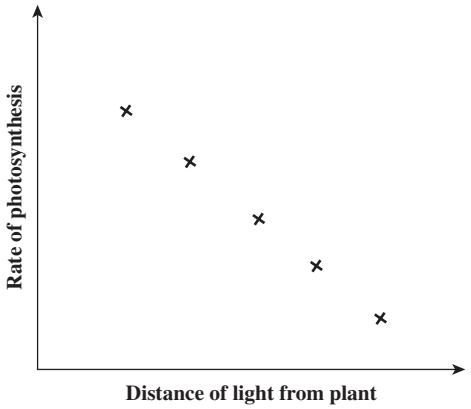


Figure 3.13 The line drawn in **b** is an example of a good line of best fit. **c** and **d** are not good lines of best fit; in **c** the line is too high and in **d** the line has the wrong gradient (steepness).

Practice question 19

A student plotted the results from a photosynthesis investigation.

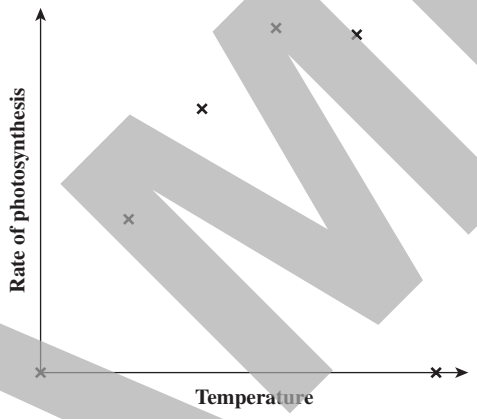


Draw a best-fit line on the graph.

Practice question 20

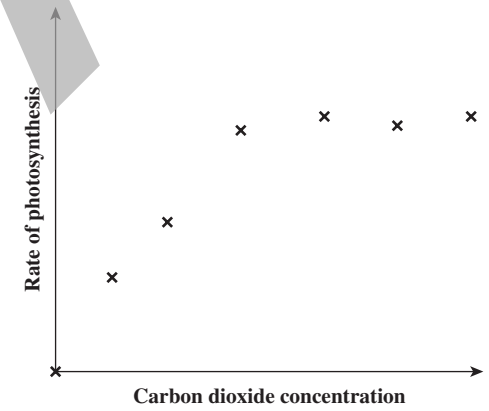
The graph below shows the results from an investigation into how temperature affects the rate of photosynthesis.

Draw a curve of best fit on the graph.



Practice question 21

Draw a curve of best fit on the graph below.



Further question

The pulse rate of a person was measured as they took part in some different activities.

The table shows the results.

Activity	Pulse rate/beats per minute
sitting	67
walking slowly	75
running	98
climbing stairs	89

On the graph paper below draw a bar chart of the results.