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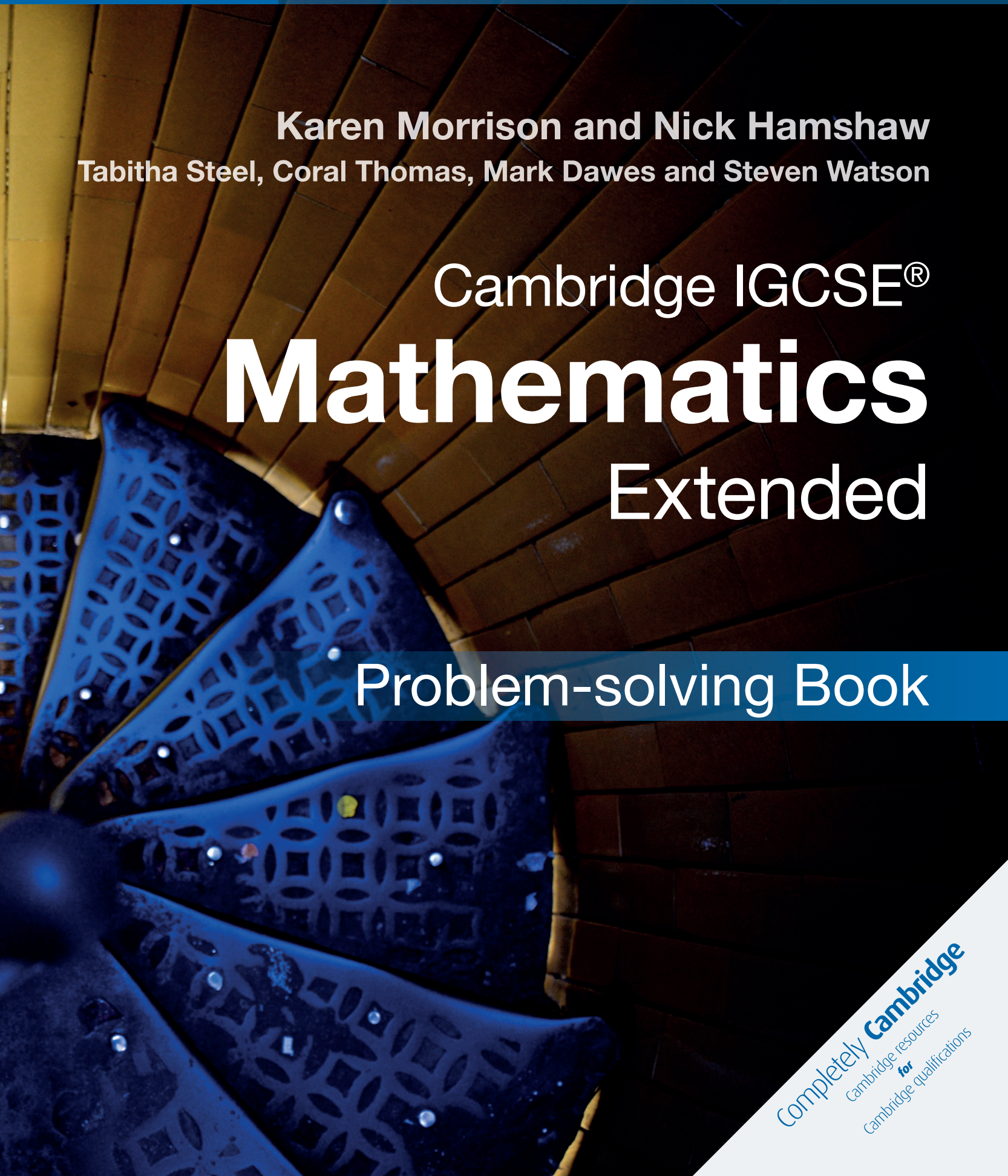
Cambridge IGCSE®
Mathematics
Extended

Problem-solving Book



Chapter 1
Draw a diagram

You might have heard the saying 'a picture is worth a thousand words'. This means that a picture can show a lot of information without using language. Different types of graphs, Venn diagrams, maps, scale drawings, tree diagrams and two way tables are all mathematical 'pictures' that can show mathematical information in a clear way.

Drawing a picture is a very useful problem-solving strategy. You can use pictures to help you 'see' a problem and to work out what you need to do to solve it.

When you are given a word problem, you can 'translate' it into a more visual form (a diagram, graph, rough sketch or table) to help you see the problem more clearly. You can also use your picture to organise the information you are given and work out what you need to do to solve the problem.

For some problems a diagram will be provided. If so, remember that you can write on the diagram and add information to it to help you solve the problem.

When no diagram is provided you can draw your own.

Decide what type of picture will be most useful. Then draw a clear diagram that is large enough to work on. You can use different colours and highlighters to make it easier to see what you are doing. Rough sketches are acceptable but your sketch should look like the thing it describes. If there is a triangle in the problem, then your shape should be a triangle. If there is supposed to be a straight line, then your line should be straight. The actual sizes of sides and angles are not important in a rough sketch.

Label your diagram. If there is information provided in the question (such as the lengths of sides, or the sizes of angles) then write these on your diagram. This will often help when you are solving a problem.

Add new information that you work out. When you work out something new, add this to the diagram too.

So, in summary:

- draw a clear diagram
- label it
- add new information that you work out.

Here are three examples where drawing diagrams could help you:

Problem 1.1: A canteen offers a 'meal deal' that allows customers to choose a main course of fish, chicken or vegetables and a side order of either rice, fries, noodles or salad.

How many different meal combinations can you choose?

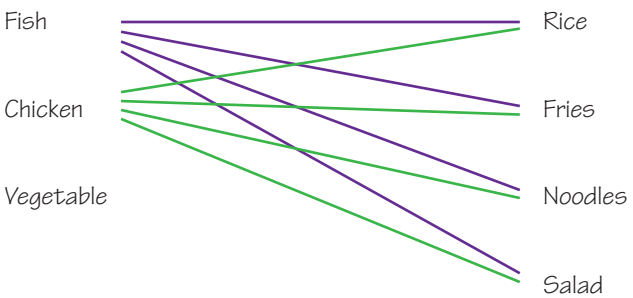
Tip

When we use the word 'diagram' here, we mean any visual representation of a problem. This can include rough sketches, graphs, number lines, tree diagrams, possibility diagrams, two way tables and Venn diagrams.

Tip

Drawing is a useful strategy to consider for problems involving combined probabilities, sets, loci, area and perimeter, ratio and proportion, fractional sharing, growth and decay, vectors, transformations, angles, distances and statistics.

You could work systematically and create a list, but a diagram would also help.



The diagram above shows all the options and the lines show some of the possible combinations.

There are four purple lines from fish to the four side options and four green lines from chicken to the four side options. This shows you that there are four choices for each main. You don't need to draw in the other lines to work out that there are 12 possible combinations.

You could use a possibility diagram like the one below to solve this problem.

Each tick, or each cell on the grid, represents one possibility. There are 12 ticks, so there are 12 possible meal combinations.

	Rice	Fries	Noodles	Salad
Fish	✓	✓	✓	✓
Chicken	✓	✓	✓	✓
Vegetable	✓	✓	✓	✓

Problem 1.2: A rectangle has sides of 10 cm and 8 cm to the nearest centimetre.

a What are the limits of accuracy for the area of this rectangle?

b What is the difference between the minimum and maximum values for:

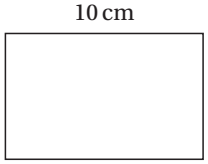
i the lengths of the sides

ii the area?

Draw a rough sketch of a rectangle. Label the sides and find the upper and lower bound of each measurement. This is the error interval.

$9.5 \leqslant L < 10.5$

10 cm



6 cm $\quad 5.5 \leqslant W < 6.5$

Tip

It can be useful to use a ruler to draw straight lines, even in sketches.

Sketch the smallest and greatest rectangles and find the area of each.

$A = 9.5 \times 5.5$
$= 52.25 \text{ cm}^2$

$A = 10.5 \times 6.5$
$= 68.25 \text{ cm}^2$

Write the values as an error interval for the area, A, of the rectangle using the correct notation.

a The limits of accuracy for the area are $52.25 \text{ cm}^2 \leqslant A < 68.25 \text{ cm}^2$

Your sketches show that the difference between the minimum and maximum values of the length and width is 1 cm.

You can subtract to find the difference between the minimum and maximum area.

b i 1 cm **ii** 16 cm^2

Here is an example where a table with highlighting is useful:

Problem 1.3: Amman says, "If I write out numbers in rows of six, all of the prime numbers will either be in the column that has 1 at the top, or in the column that has 5 at the top".

Can you tell if he is right?

You need to have some numbers to look at here so a diagram will be important.

Highlight a few prime numbers.

The table shows that the numbers in each column increase by six as you move down.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

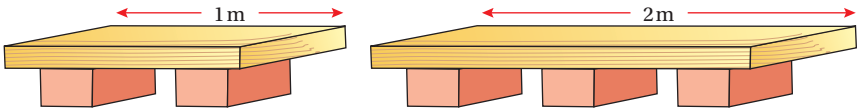
You know that 2 is the only even prime number. So you can eliminate all numbers in the 2nd, 4th and 6th columns, except for the 2 in the first row. (Adding six to an even number will always give an even answer.)

3
9
15
21
27
33

Adding six is the same as adding two threes, so all the numbers in the column with 3 at the top must multiples of 3 and therefore not prime, except for the 3 in the first row.

This means that besides the first row, any prime numbers must be in the first or fifth columns so Amman is right.

In the local cement factory, the cement bags are placed on pallets made of planks of wood and bricks.



The number of bricks needed to make the pallet is calculated as 'one more than the length of the plank in metres'.

a What length of pallet uses five bricks?

b If the pallet is 7 m long, how many bricks are used in it?

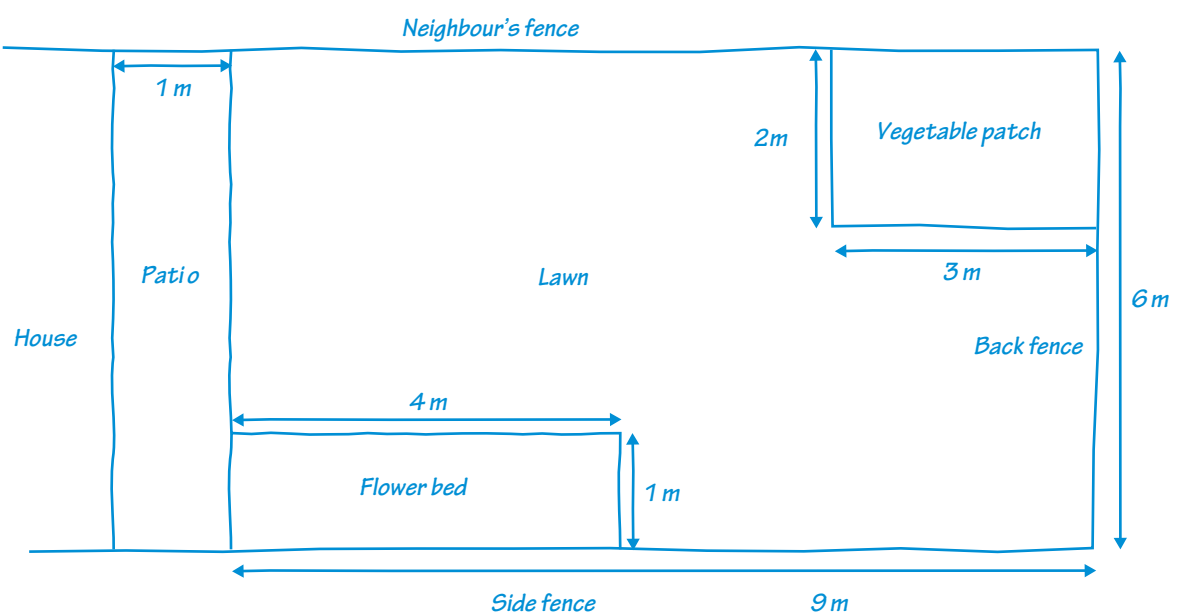
The factory needs pallets with a total length of 15 m for the next batch of cement. It has planks of wood that are 4 m long and 3 m long.

c What combinations of planks can they have?

d How many bricks would be needed for each combination?

Sanjita wants to plant a cherry tree in her garden. She needs to make sure there is a circular area of lawn with diameter 3m around the base of the tree, so that all of the fruit will fall onto the lawn area.

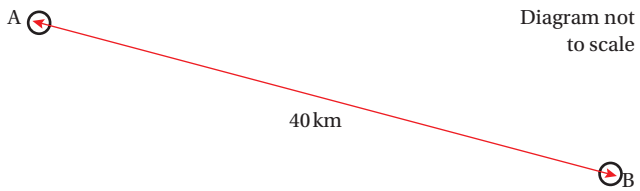
Here is a sketch, not drawn to scale, of Sanjita's garden.



Where could the tree be planted to meet her conditions?

3

The diagram represents towns A and B in a mountainous region.



The mountain rescue helicopters from both towns will always be sent to rescue any casualty within a radius of 25 km of town A or town B. The fire and rescue team from town B will travel to any accident scene closer to town B than town A.

Shade the region that the helicopters and town B's fire and rescue team will both cover.

4

A rectangle has length $(2x + 3)$ and width $(x - 1)$.

a Write an expression for the perimeter of the rectangle.

b Write an expression for the area of the rectangle.

The area of the rectangle is 250 cm^2.

c How long is the longest side?

d What is the perimeter of the rectangle?

Tip

253 = 23 × 11

5

The probability that Hamza catches the 6.30 am train to the city is 0.7.

If he misses the train he will be late for work.

The probability the train will be late is 0.15.

If the train is late he will be late for work.

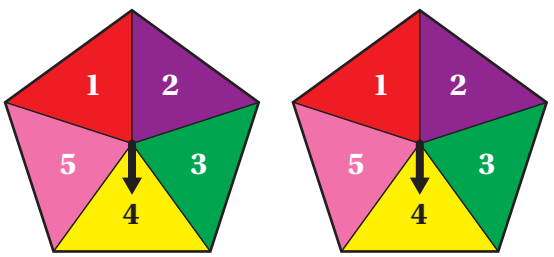
What is the probability Hamza will be on time for work on a particular day?

Tip

What type of diagram might help here?

6

Two five-sided spinners are numbered 1 to 5. When the arrows are spun, your total score is calculated by adding the two numbers that the spinners land on.



a Draw a suitable diagram to show all possible outcomes when spinning these spinners.

b What is the highest score you could get?

c What is the probability of getting a total score of 8?

7

The vertices of a quadrilateral are A, B, C and D.

A has coordinates (2, 1).

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

a Write a column vector for $\overrightarrow{CD}$.

b Compare $\overrightarrow{CD}$ with $\overrightarrow{AB}$. What do you notice? Can you explain?

c What type of quadrilateral is ABCD?

Tip

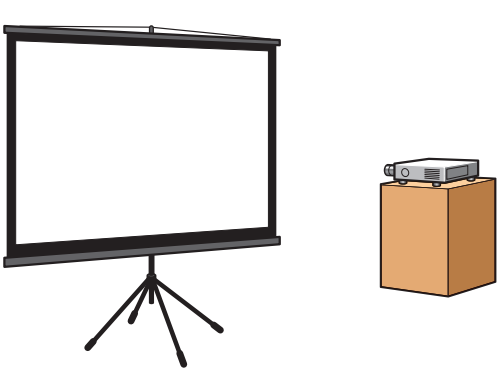
Use squared paper to draw your diagram.

8

A projector is placed 1 m from a screen. When the projector is turned on, the image produced is only 20 cm high.

How far back should the projector be moved to produce an image that exactly fills the screen, which is approximately 1.5 m in height?

(Assume that no other adjustments are made to the projector.)



9

A factory manager planned to install a new hot drinks machine for the factory workers. He thought tea would be the most popular hot drink.

The workers did a survey to check what the preferred hot drink was among them. Each person could choose one drink from hot chocolate, tea or coffee.

Eight women wanted hot chocolate.

A total of 16 workers wanted tea, of which seven were men.

10 men and 12 women chose coffee.

There were 25 men in total.

Was tea the most popular hot drink?

Tip

What type of diagram might be helpful?

10

A ship sails 17.6 km on a bearing of 270° and then 15.4 km due south.

What is the shortest distance back to where it started?

11

Maria needs to make a long-distance journey. She is looking for the cheapest car hire.

Whacky Wheels has a standard charge of $35, then 15¢ for every kilometre driven.

Wheelies Rentals has a charge of 23¢ per kilometre travelled, but no standard charge.

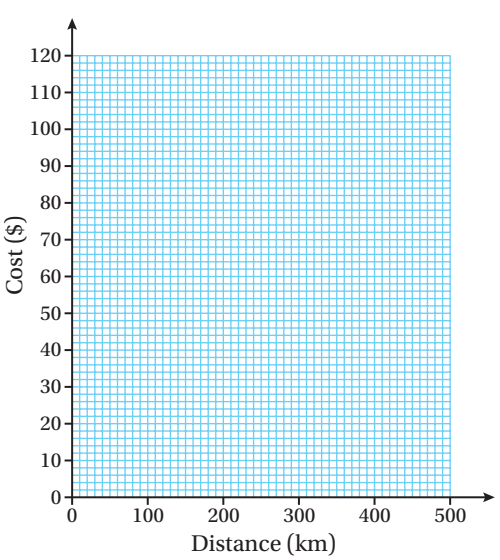
a Complete the charges graph for both car hire companies.

b Maria thinks the return journey will be 300 km. Which company would be cheaper to use?

c Maria made a mistake in her route plan and the return journey was 500 km. How much money would Maria have saved by using the other hire company?

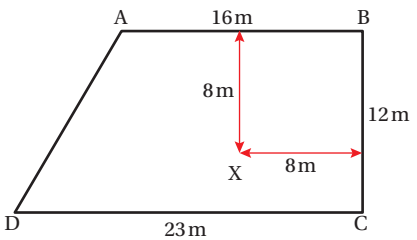
Tip

In this question you can use the axes that are given to help you draw the diagram.



12

ABCD is a field surrounded by fences AB, BC, CD and DA.



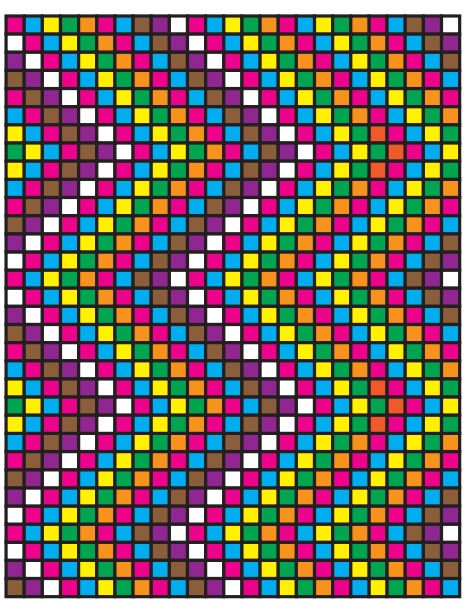
A goat is tied to the spike X on a rope measuring 3 m.

A bull is tied by an 8 m rope to the top of post A.

Find a route from corner D to corner B that would avoid both the bull and the goat.

13

This patchwork quilt is made from scraps of fabric.



Each patch is $(2x - 3)$ cm long and $(x + 3)$ cm wide. The area of the completed quilt is 2.8 m^2.

a There are 25 patches in each row. Write a possible expression for the width of the quilt.

b There are 32 patches in each column. Write a possible expression for the length of the quilt.

Tip

The diagram in the question is very detailed. Could a simpler diagram help?

c Write an expression for the area of the quilt, in the form $m(ax^2 + bx + c)$ where m is a constant.

d What are the dimensions of each patch? Give your answers in centimetres.

Tip

$2.8\,m^2 = 28\,000\,cm^2$. It might be easier to work in centimetres.

14 ✪✪☆

A square-based food container has a capacity of $1440\,cm^3$.

The base of the container has length x cm.

a Write an equation for the height of the container in terms of x.

The inside of the container (base and four sides) is to be lined with grease-proof paper with no overlaps.

b What is the formula, in terms of x, for the area of grease-proof paper needed?

c If the height of the container is 10 cm, what is the area of the base?

d What is the area of grease-proof paper needed?

15 ✪✪☆

A rectangular swimming pool is surrounded by a path made of mosaic tiles. The width of the path is x. It cost \$3196.80 to have the path tiled, at a rate of \$30 per square metre.

The pool itself measures 35 m by 30 m.

a Write an expression for the area of the tiled path in terms of x.

b Find the width of the path to the nearest centimetre.

16 ✪✪☆

Serrianne has taken up golf and goes to practise at the golf range twice a week. She uses one bucket of balls each time. In every bucket of 25 balls there are always 3 yellow balls; the rest are white.

Serrianne hits one ball (chosen at random) at a time.

a What is the probability that the first 3 balls she uses will all be yellow?

b What is the probability that the first 3 balls she uses will all be white?

c Calculate the probability that the first 3 balls Serrianne uses are a mixture of two yellows and one white.

17 ✪✪☆

To make the journey to work Abu must drive through two sets of traffic lights.

The probability of the first set being green is 0.7. If the first set is green, the probability of the second set also being green is 0.8. But if the first set is not green, the probability of the second set being green is 0.4.

a What is the probability that Abu does not have to stop on his journey to work tomorrow?

b What is the probability that Abu only has to stop once on his journey to work tomorrow?

18 ✪✪☆

Kalima and Jiao are very competitive and often have badminton and squash matches. The probability of Kalima winning at badminton is 0.85 and the probability of Kalima winning at squash is 0.35.

a What is the probability that the next time they play both matches, Kalima wins both?

b What is the probability that Kalima loses at badminton but wins at squash?

c What is the probability that both girls win one match each?

19 ✪✪☆

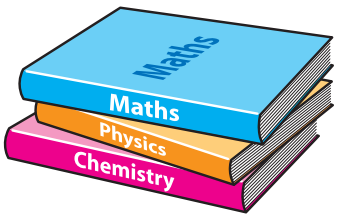
On a commercial flight to Tanzania the passengers were questioned about their malaria precautions. Only 70% of the passengers had obtained and started a course of anti-malaria tablets. The chances of getting malaria are $\frac{1}{200}$ if you take the tablets but $\frac{1}{50}$ if you are not taking the tablets. What is the probability that one passenger selected randomly will contract malaria?

Tip

What type of diagram would be helpful?

20 ★★☆

The owner of a bookshop carried out a survey to find the most popular school subjects in Year 10 to help decide how many revision guides to stock. A total of 200 students were asked whether they were studying Chemistry, Physics or Maths.



43 of the students surveyed did not study any of these 3 subjects.

A total of 92 were studying Chemistry.

There were 23 studying both Chemistry and Maths, but not Physics.

There were 19 studying both Physics and Maths, but not Chemistry.

29 were only studying Physics, and there were a total of 74 who studied Physics.

53 of the students studied 2 of these 3 subjects.

a Display the information in an appropriate diagram.

b If one person was chosen at random, what is the probability they only studied maths?

c If one person was chosen at random, what is the probability they studied at least two of the subjects?

21 ★★☆

The point A has coordinates (2, 2).

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Tip

Draw a sketch of the shape.

a Find a possible pair of coordinates for D, if ABCD is an isosceles trapezium.

b Write $\overrightarrow{AC}$ as a column vector.

c Find the coordinates of E, if $\overrightarrow{AE} = 4\overrightarrow{BC}$.

d Using these coordinates for E, write $\overrightarrow{BE}$ as a column vector.

22 ★★☆

a Amira has a challenge for her classmate, Janet:

"I'm thinking of a triangle...

It has a right angle. It has one angle of 40°. It has one side that has a length of 5 cm.

Draw my triangle."

i Accurately construct a triangle that satisfies Amira's conditions.

ii Demonstrate that there is more than one triangle that Amira could be thinking of.

iii How could Amira alter her challenge so that only one triangle is possible?

b Janet comes up with a challenge for Amira:

"I'm thinking of a triangle...

It has one side of length 4 cm. It has another side of length 7 cm. The angle in between these two sides is 55°.

Draw my triangle."

i How many triangles satisfy Janet's conditions? Explain your answer.

ii Find the length of the third side.

Tip

To construct an accurate triangle you need to use a ruler, protractor and a pair of compasses.

Tip

Before you start your accurate construction, make some rough sketches to show the positions of the sides and angles you are given.

23 ★★☆

Raj took a photo of his mother on holiday. Later, when Raj looked at the picture, he saw that his mother seemed to be the same height as a hill in the background.

Raj stood approximately 3 m away from his mother when he took the photo, and they were about 2 km away from the hill. Raj's mother is 15 cm taller than him.

Approximately how high is the hill?

24 ★★☆

When enlarging photographs, the increase in width and length must be directly proportional to each other or the photos will be distorted.

A photo has width 40 cm and length 55 cm.

a An enlargement of this photo has width 112 cm. Find the length for this photo poster.

b Another enlargement of the original photo has length 148.5 cm. What must the width be for this enlargement?

c A third photo with length 15 cm and area 127.5 cm^2 is enlarged to a poster of width 25.5 cm. What is the area of this poster?

25 ★★☆

Suki and Fleur do some swimming every morning. They swim a total of 45 lengths each. They always start together but never finish together. They swim at different speeds for different swimming strokes.

Suki always swims 45 lengths of breaststroke in 30 minutes, completing each one at the same speed.

Fleur always does 30 lengths of front crawl in the first 12 minutes, then the remaining 15 lengths at a speed of one length per 40 seconds.

a After ten minutes, how many lengths has Suki completed?

b How long does it take Fleur to complete her final 15 lengths?

c What is Suki's speed in lengths per minute?

d How long must Fleur wait for Suki to finish?

e Roughly, on average, how many lengths does Fleur swim each minute?

f If Suki continued swimming for another 10 minutes, in theory how many lengths should she complete in total? Explain why this figure might not be correct.

26 ★★★

You can use the three transformations listed below:

A Reflect in the line $y = x$

B Translate by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

C Enlarge by scale factor $\frac{1}{2}$ about the point (2, 3)

a Carry out all three transformations, in order, on a starting shape of your choice.

b How does the resulting image change if the transformations are applied in reverse order? C ⟶ B ⟶ A

c How many different final images could be produced by changing the order in which the three transformations are applied?

Tip

You will find this question easier if you try it out. Think about how you can make it simpler by choosing shapes and side lengths that make the enlargement easier.

27 ★★★

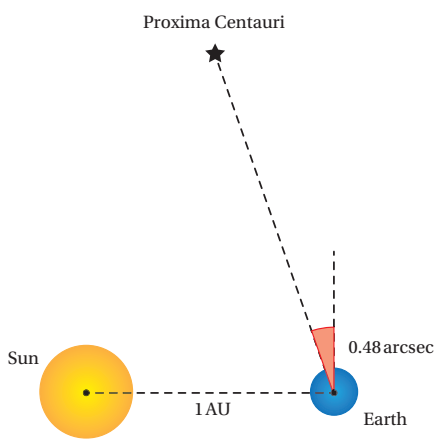
Two of the vertices of an equilateral triangle are located at points with coordinates (0, 0) and (6, 0).

a Work out the possible coordinates of the third vertex.

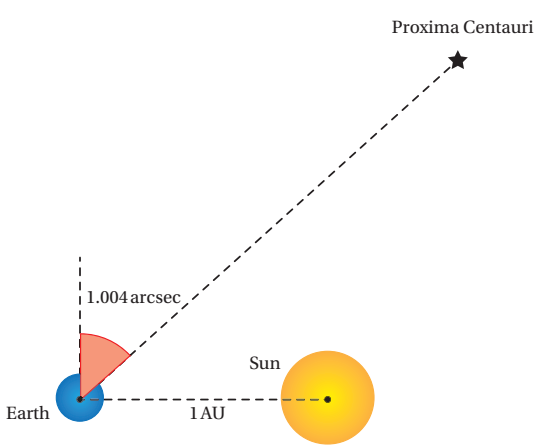
b If two of the vertices of a different equilateral triangle are located at (−3, 2) and (5, −4), what is its area?

28 ★★★

An astronomer wants to calculate the distance to one of our closest stars, Proxima Centauri. To do this, she takes two angle measurements, six months apart. The two angles measured by the astronomer are shown in the diagrams below.



Six months later...



Key facts:

$1\text{ AU} \approx 1.5 \times 10^8\text{ km}$

$1\text{ arcsec} = \left(\frac{1}{3600}\right)^\circ$

Use the astronomer's measurements to calculate the approximate distance to Proxima Centauri.

29 ★★★

Find the area of the triangle enclosed by the x-axis and the straight lines with equations $3x + 2y = 15$ and $y - 2x = 4$.

Tip

Work out where the lines cross the axes and use these coordinates to help you draw a diagram.
It is helpful to consider one more point when calculating the area of the triangle. How will you find the coordinates of this point?

30 ★★★

Rectangle ABCD has width 1 cm and length k cm, where k is greater than 1. AB = 1 cm and BC = k cm. The rectangle is divided into a square and a smaller rectangle by drawing a line parallel to the side AB. The smaller rectangle is mathematically similar to the rectangle ABCD. Calculate the value of k.

Tip

Draw a diagram. You need to be able to work out the length and width of the smaller rectangle.

31 ★★★

Two circles, one larger than the other, have the same centre. A chord is drawn joining two points on the circumference of the larger circle. This chord is also a tangent to the smaller circle. The chord has length 20 cm. Find the area of the ring enclosed between the circles.

Give your answer as an exact multiple of π.

Tip

You do not need to know the radius of either circle to answer this question, but you do need to find a relationship between the two radii. What can you draw on the circle to help you?

Worked solutions

1 Draw a diagram

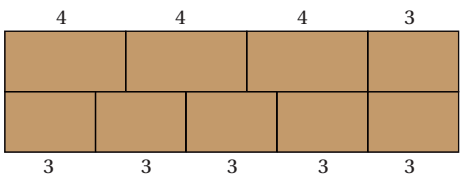
1 **a** 4 metres

One less than five bricks = 4

b 8 bricks

Length + 1 = number of bricks

c



Any combination of 4 and 3 to total 15.

4 m + 4 m + 4 m + 3 m

3 m + 3 m + 3 m + 3 m + 3 m

d 19 bricks

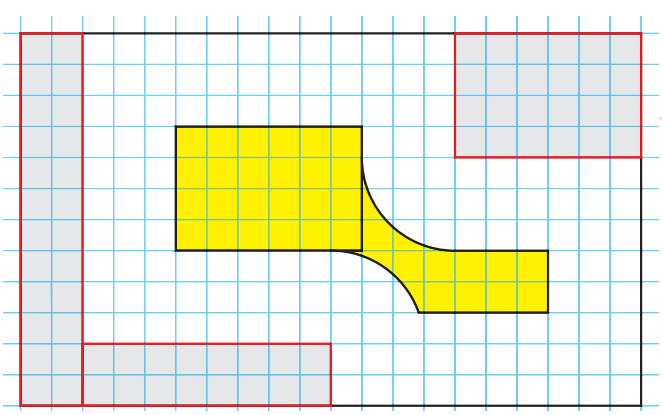
20 bricks

4 m + 4 m + 4 m + 3 m would need:
5 + 5 + 5 + 4 bricks (for each plank there is one more brick than its length, in metres)

or

3 m + 3 m + 3 m + 3 m + 3 m would need:
4 + 4 + 4 + 4 + 4 bricks

2 The area into which the fruit can fall has a diameter of 3 m and therefore a radius of 1.5 m. This means the tree can be planted anywhere 1.5 m away from the edge of the lawn.

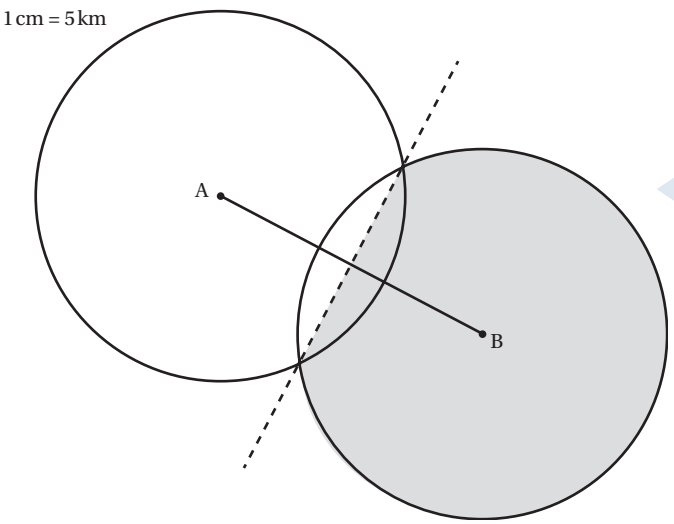


You can choose any scale you like.

To show where the cherry tree can go there are straight lines 1.5 m away from the edge of the lawn and arcs (parts of circles) at the corners of the flower bed and the vegetable patch.

This diagram has a scale of two squares to 1 m. The yellow shaded area is where the cherry tree can be planted.

3 The helicopters cover 25 km from each town, so two circles are needed. The fire brigade covers the section closer to B than A, so we need the locus of points equidistant from both towns.



Here the scale is 1 cm = 5 km (you might have chosen a different scale).

Start with a line of 8 cm (to represent the 40 km distance between the towns).

With compasses draw a circle centre B and radius 5 cm.

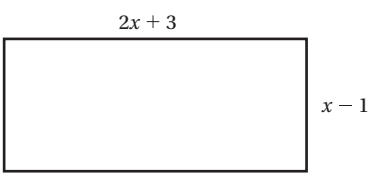
With compasses draw a circle centre A and radius 5 cm.

The locus of points equidistant from B and A is the perpendicular bisector of line AB.

Shade the region that is closer to B than A and covered by the helicopter.

---

4 **a**



An expression for the perimeter is $6x + 4$

b An expression for the area is $(2x + 3)(x - 1)$, which can be expanded and simplified to give $2x^2 + x - 3$.

c $2x^2 + x - 253 = 0$, which can be factorised to give $(2x + 23)(x - 11) = 0$

$x = -11.5$, or $x = 11$

x cannot be negative (because that would mean at least one of the sides of the rectangle would be negative), so $x = 11$ and the two sides are 25 cm and 10 cm.

The longest side is therefore 25 cm.

d The perimeter is 70 cm.

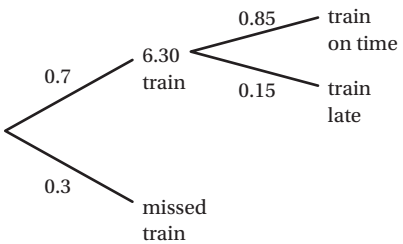
To work out the perimeter of a rectangle, you need to add up the sides. To work out the area you multiply the two sides.

A diagram will help.

Make an equation and expand and simplify it.

You need to solve $2x^2 + x - 3 = 250$.

5



$0.7 \times 0.85 = 0.595$

A tree diagram might be useful here.

6 **a**

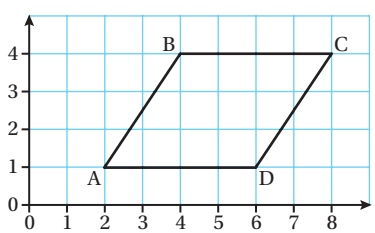
+	**1**	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

b 10

c $\frac{3}{25}$

Here is a diagram that will work well.

7



a $\overrightarrow{CD} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

b $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

The length AB is the same as the length CD but the vectors are in opposite directions.

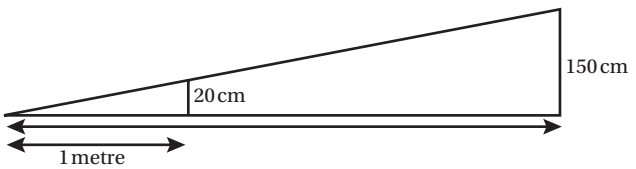
c ABCD is a parallelogram – a four-sided shape with two sets of parallel sides.

A diagram of the situation helps you see what is going on.

BC is $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and AD is $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ so BC is parallel to AD

AB is $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and DC is $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ so AB is parallel to DC

8



The scale factor of the enlargement is
150 cm ÷ 20 cm = 7.5

1 m × 7.5 = 7.5 m

The projector needs to be 7.5 m away for the image to fill the screen. So, it has to be moved back another 6.5 m.

A good diagram shows that this is a question about similar triangles.

9

	Hot chocolate	Tea	Coffee	Total
Women	**8**	9	**12**	29
Men	8	**7**	**10**	**25**
Total	16	**16**	22	54

Of the 54 workers surveyed, 22 preferred coffee, 16 preferred tea and 16 preferred hot chocolate.

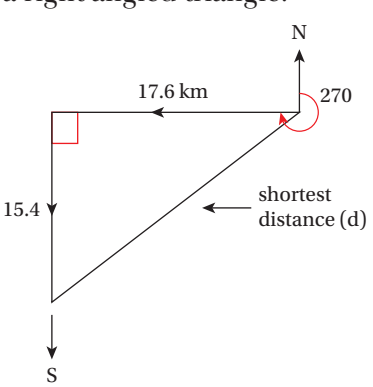
Tea is not the most popular drink. Coffee is the most popular hot drink among the workers and hot chocolate is equally as popular as tea.

A diagram like this 2-way table will help here.

The bold numbers are given in the question and the rest can be worked out.

---

10 The shortest distance (d) is the hypotenuse of a right-angled triangle.



$(17.6)^2 + (15.4)^2 = d^2$

$309.76 + 237.16 = 546.92$

$d = \sqrt{546.92}$

d = 23.39 km (correct to 2 d.p.)

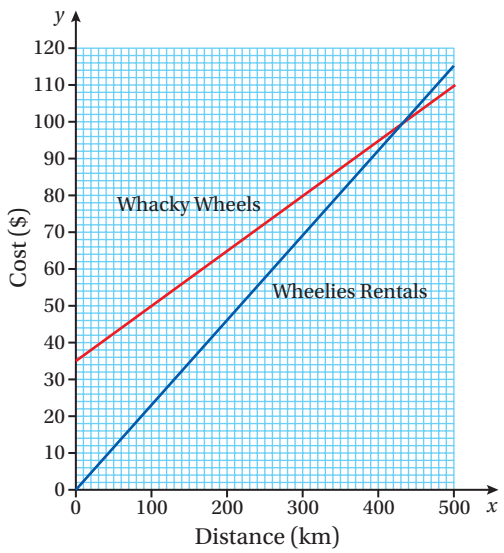
Draw a sketch showing the route.

A bearing of 270° is due west.

The sketch shows that the route forms a right angle, so you can use Pythagoras' theorem to find the distance needed.

---

11 **a**



The graph is helpful in answering the rest of this question.

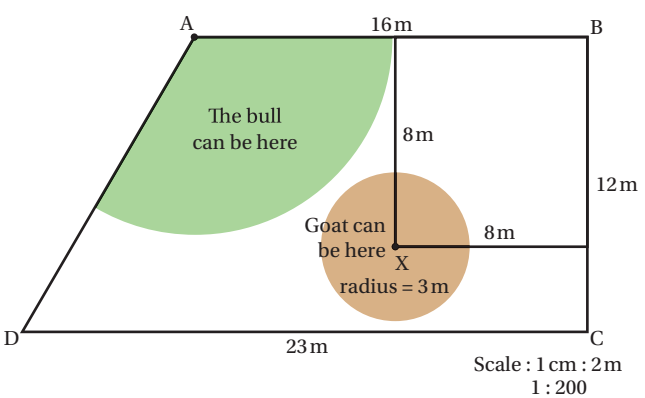
b Wheelies Rentals

c $5

Whacky Wheels: $35 + $75 = $110

Wheelies Rentals: 5 × $23 = $115

12



A good way to solve this is to draw a scale diagram. You could use a scale of 1 cm to 2 m.

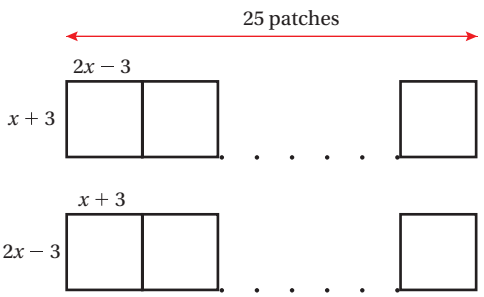
On the scale diagram, construct:

- A circle of radius 1.5 cm about X to show where the goat can go.
- A locus of radius 4 cm about A between the fence lines AD and AB where the bull can roam.

Any path between D and B that does not enter either of the shaded areas would be safe to use.

---

13 **a**



There are 25 patches in each row.

The width could be $25(x + 3) = 25x + 75$

or it could be $25(2x - 3) = 50x - 75$

b $64x - 96$ or $32x + 96$

$32(2x - 3)$ or it could be $32(x + 3)$

c The two possible expressions for the area are $(25x + 75)\ (64x - 96)$ and $(50x - 75)\ (32x + 96)$.

These are equal because they are both equivalent to working out $25(x + 3) \times 32(2x - 3)$.

This is $800(2x^2 + 3x - 9)$.

d $2.8\,m^2$ is the same as $2.8 \times 100\,cm \times 100\,cm = 28000\,cm^2$.

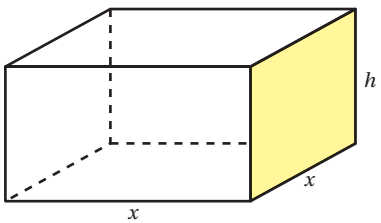
The equation is therefore: $800(2x^2 + 3x - 9) = 28000$

Dividing through by 800 gives: $2x^2 + 3x - 9 = 35$

$2x^2 + 3x - 44 = 0$ can be factorised to give $(2x + 11)\ (x - 4) = 0$, so $x = {}^{-}5.5$ (which isn't possible for this scenario) or $x = 4$.

The dimensions of the patches are: $2x - 3$ by $x + 3$ and when $x = 4$ this gives 5cm by 7cm.

14 **a**



It's useful to draw a diagram of the container.

The volume is $x^2 \times h$. This equals 1440, so:
$x^2h = 1440$, which means $h = \frac{1440}{x^2}$

b The base has area x^2.
Each of the side walls (shown in yellow on the diagram) has area hx, which is $x \times \frac{1440}{x^2}$, which simplifies to give $\frac{1440}{x}$.
There are four of these, so the total area of paper is $x^2 + \frac{5760}{x}$

c Area = 144 cm^2

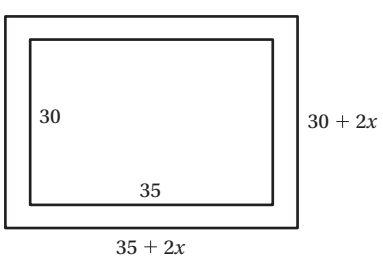
$1440 = x^2 \times 10$

So $144 = x^2$

$x = 12$

d 624 cm^2

15



The diagram shows the top view of the pool and the path.

a $(35 + 2x)(30 + 2x) - 35 \times 30 = 130x + 4x^2$

You could work out the area of the outer rectangle and then subtract the area of the inner rectangle.

b $3196.80 \div 30 = 106.56$

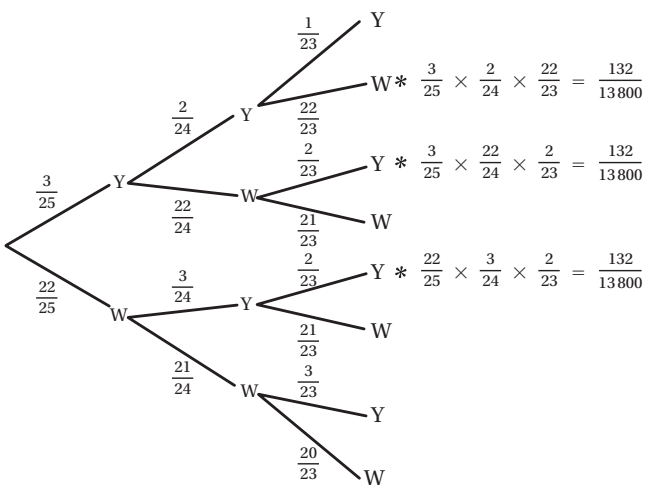
Square metres of border.

$4x^2 + 130x = 106.56$

Solve this equation.

$x = 0.8$ m

16



A probability tree diagram is probably the most useful diagram here.

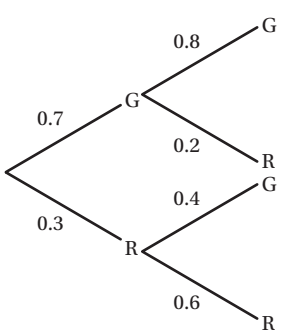
a P (3 yellow) $= \frac{3}{25} \times \frac{2}{24} \times \frac{1}{23} = \frac{6}{13\,800} = \frac{1}{2300}$

b P (3 white) $= \frac{22}{25} \times \frac{21}{24} \times \frac{20}{23} = \frac{9240}{13\,800} = \frac{77}{115}$

c $\frac{132}{13\,800} + \frac{132}{13\,800} + \frac{132}{13\,800} = \frac{396}{13\,800} = \frac{33}{1150}$

The three routes labelled on the diagram all work, so you need to add them.

17



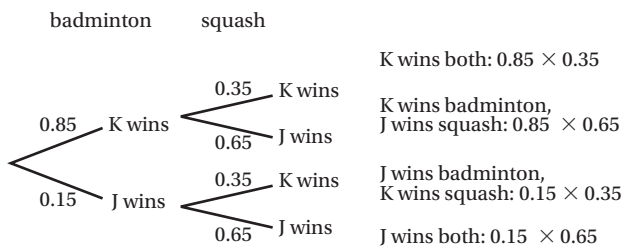
A tree diagram might help.

a $0.7 \times 0.8 = 0.56$

b $(0.7 \times 0.2) + (0.3 \times 0.4) = 0.26$

Two outcomes: green then red (GR) and red then green (RG).

18



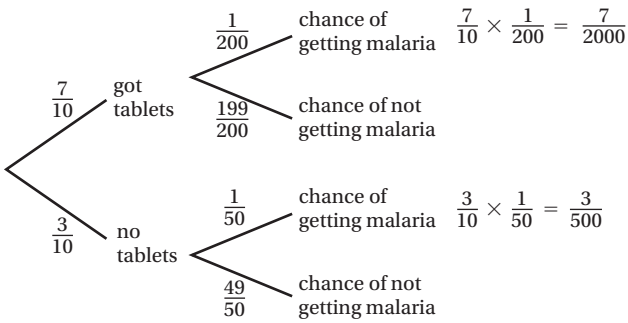
Here is the tree diagram.

a $0.85 \times 0.35 = 0.2975$

b $0.15 \times 0.35 = 0.0525$

c $0.85 \times 0.65 + 0.15 \times 0.35 = 0.605$

19



A tree diagram can be helpful.

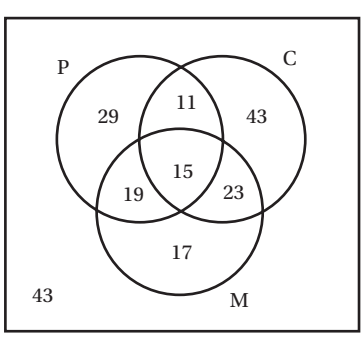
$\frac{7}{10} \times \frac{1}{200} = \frac{7}{2000}$

$\frac{3}{10} \times \frac{1}{50} = \frac{3}{500}$

$$\frac{7}{2000} + \frac{3}{500} = \frac{19}{2000} = 0.0095$$

---

20 **a**



A Venn diagram is likely to be useful here.

53 study two subjects – so 11 study Physics and Chemistry but not Maths.

74 study Physics, so 15 study all three.

92 study Chemistry, so 43 study only Chemistry.

There are 200 students so 17 study only Maths.

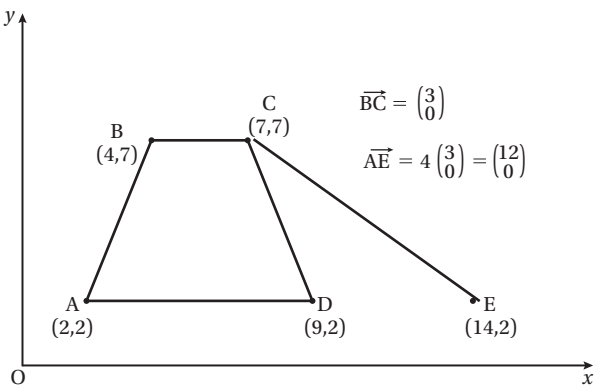
b $\frac{17}{200}$

c $\frac{68}{200}$

11 studied Chemistry and Physics, 19 studied Physics and Maths, 23 studied Chemistry and Maths, 15 studied all three.

Total: 68 of 200 students.

21 **a**



An isosceles trapezium has one line of symmetry, so D could be (9, 2) as C to D will be 2 units to the right and 5 units downwards.

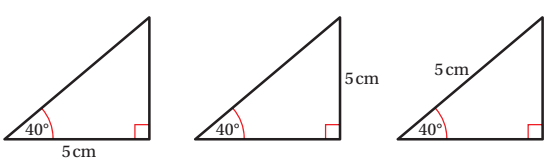
b $\overrightarrow{AC} = \begin{pmatrix}5\\5\end{pmatrix}$

c E (14, 2)

d $\overrightarrow{BE} = \begin{pmatrix}10\\-5\end{pmatrix}$

---

22 **a** **i, ii**



There are three possible triangles that fit Amira's description.

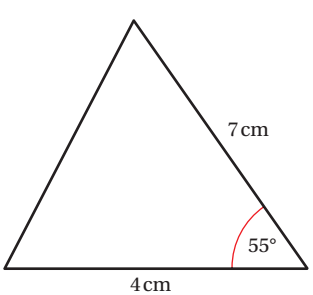
iii She could say that the hypotenuse is 5 cm.

Other answers are possible too.

For only one triangle to be possible, Amira must make sure that her conditions follow one of the conditions of congruence:

SSS, SAS, ASA, AAS or RHS

b **i**



There is only one triangle that satisfies Janet's conditions because she has given SAS.

A scale drawing will enable you to measure the length of the third side: 5.7 cm to the nearest mm.

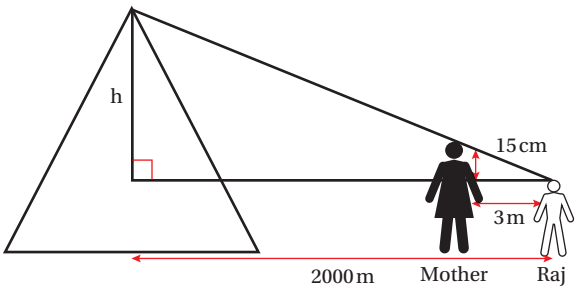
ii $a^2 = b^2 + c^2 = 2bc\cos A$

$a^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \times \cos 55°$

$a = 5.734\ldots$

A more accurate method to find the length of the third side is to use the cosine rule.

23



This is a complicated scenario. A diagram can help you see what is going on.

The ratio of height: distance is the same for both Raj's mother and the hill.

0.15 : 3

h : 2000

$0.15 \div 3 \times 2000 = 100$ m

Divide by 3 and multiply by 2000.

The hill is about 100 m tall.

---

24 **a** width : length

40 : 55

8 : 11

$112 \div 8 = 14$

$11 \times 14 = 154$ cm

You might want to draw diagrams to show what is going on.

This is the length.

b $148.5 \div 11 = 13.5$

$13.5 \times 8 = 108$ cm

This is the width.

c Width of photo = area ÷ length

$= 127.5 \div 15$

$= 8.5$ cm

First work out the width of the original photo using the length and area given in the question.

Factor of enlargement

= new width ÷ original width

$= 25.5 \div 8.5$

$= 3$

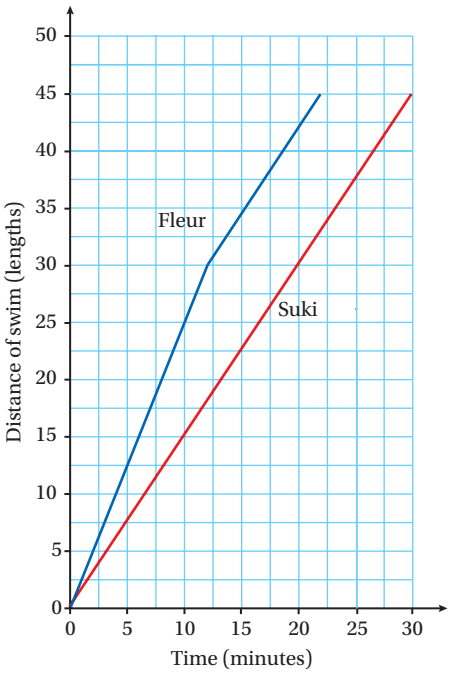
Then work out the scale factor of the enlargement by comparing the two widths.

The area will increase by a factor of $3^2 = 9$.

Area of poster = (127.5×9) cm^2

$= 1147.5$ cm^2

25



This is the sort of question where a graph can really help.

a 15

b 10 minutes

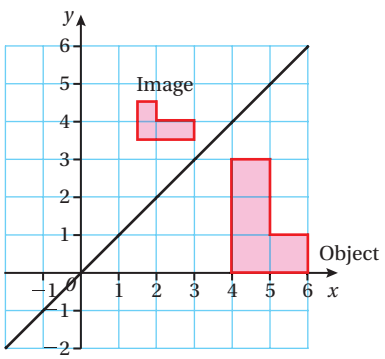
c 45 ÷ 30 = 1.5 lengths per minute

d 8 minutes

e About 2 lengths per minute

f 15 more lengths, which is 60 in total, but she might get tired and start to slow down.

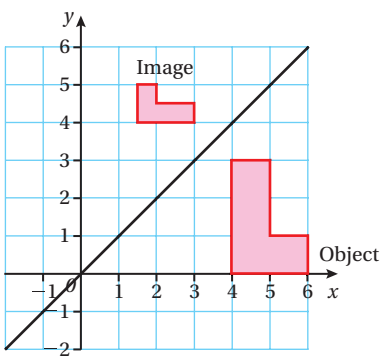
26 **a**



Choose a shape without any symmetry to easily see what is going on.

Here is an example.

b



The orientation and size of the final image are the same when the transformations are carried out in a different order, but it ends up in a different place.

c 1, 2, 3

1, 3, 2

2, 1, 3

(2, 3, 1)

3, 1, 2

3, 2, 1

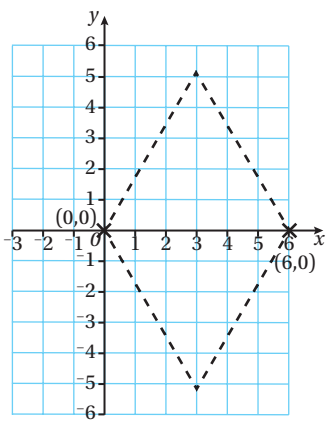
There are five different final images.

Consider all the possible arrangements of the three transformations.

You have already drawn the first and last one.

The others result in an image that is the same size and same way around, but which is in a different place, except for order 2, 3, 1 which has an image in the same place as order 1, 2, 3.

27 **a**



There are two possible locations for the third vertex.

$a^2 + b^2 = c^2$

$3^2 + b^2 = 6^2$

$9 + b^2 = 36$

$b^2 = 27$

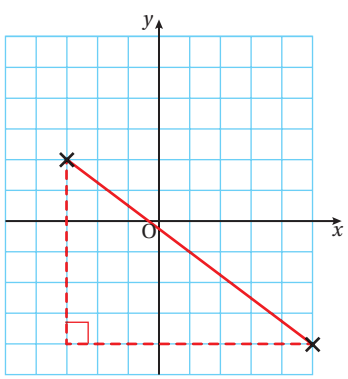
$b = \sqrt{27}$ (= 5.196…)

$(3, \sqrt{27})$ or $(3, -\sqrt{27})$

The x-coordinate is 3.

Use Pythagoras' theorem to calculate the vertical height of the equilateral triangle.

b



Here is a diagram showing the two points.

This time you only need to know the length of the sides. Pythagoras tells you the side has a length of 10.

$6^2 + 8^2 = c^2$

$c^2 = 100$

$c = \sqrt{100} = 10$

$5^2 + b^2 = 10^2$

$25 + b^2 = 100$

$b^2 = 75$

$b = \sqrt{75}$ (= 8.660…)

Area $= \frac{1}{2} \times 10 \times \sqrt{75}$

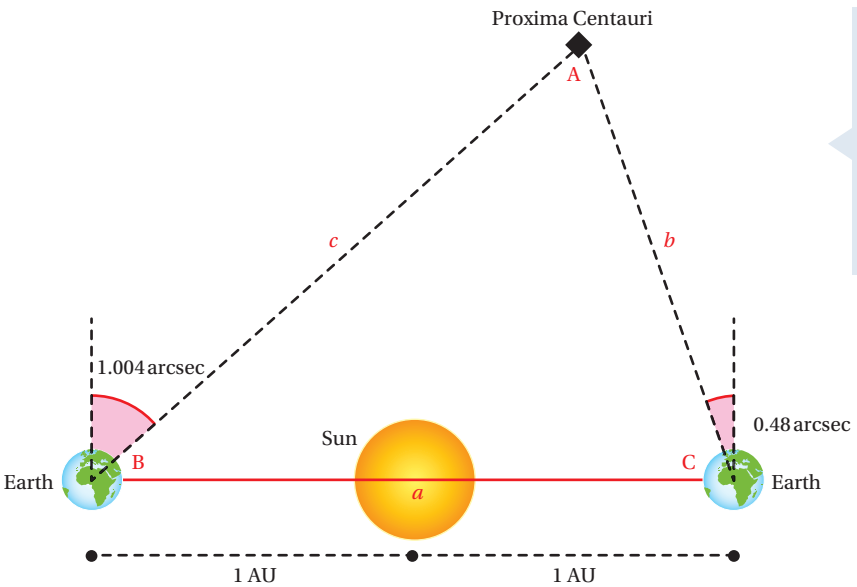
Area = 43.301…

43.3 square units

Now, if you have an equilateral triangle of side 10 you can do this.

Calculate the area of the equilateral triangle $\left(\frac{1}{2} \times \text{base} \times \text{height}\right)$

28



There are lots of aspects of this question that aren't particularly clear, so you will need to make some assumptions (which is fine as long as you explain them) and a good diagram will be a very important starting point.

It looks as if the angles are measured perpendicularly from the line through the Sun, so:

angle B is $90° - 1.004\,\text{arcsec}$

angle $C = 90° - 0.48\,\text{arcsec}$

angle A is $1.004 + 0.48 = 1.484\,\text{arcsec}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = \frac{2\text{AU}}{\sin(1.484\,\text{arcsec})} \times \sin(90° - 1.004\,\text{arcsec})$$

This gives $b = 4.169\,773\,711 \times 10^{13}\,\text{km}$

$$c = \frac{2\text{AU}}{\sin(1.484\,\text{arcsec})} \times \sin(90° - 0.48\,\text{arcsec})$$

This gives $c = 4.169\,773\,711 \times 10^{13}$

The distance is therefore about $4.2 \times 10^{13}\,\text{km}$

It is not obvious which distance is being asked for – so let's work out lengths b and c.

These answers are the same for the first 10 significant figures (which makes sense because Proxima Centauri is very, very far away compared to the diameter of the orbit of the Earth).

29 For $3x + 2y = 15$

$x = 0 \Rightarrow 2y = 15$ so $y = \frac{15}{2}$

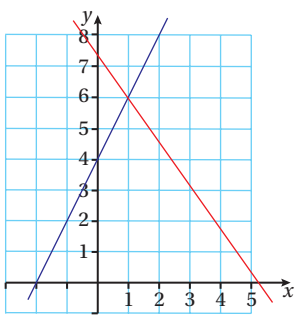
$y = 0 \Rightarrow 3x = 15$ so $x = 5$

For $y - 2x = 4$

$x = 0 \Rightarrow y = 4$

$y = 0 \Rightarrow -2x = 4$ so $x = -2$

Find the coordinates of the points where each line crosses the axes. This will help you to draw the lines.



Now draw a diagram using this information.

$3x + 2y = 15 \Rightarrow 6x + 4y = 30$

$y - 2x = 4 \Rightarrow -6x + 3y = 12$

Adding: $7y = 42 \Rightarrow y = 6$

Triangle height = 6

The y-coordinate of the point at which the lines meet will be the same as the height of the triangle. Use simultaneous equations to work out this y-coordinate. You will need to eliminate x or find x and use that to find y. Here we eliminate x.

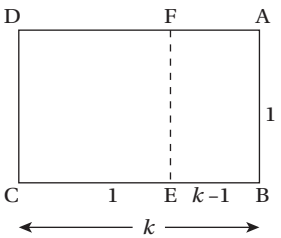
Base length = $5 - -2 = 7$

Note that the points at which the lines crossed the x-axis have been found, so you can find the length of the triangle's base.

Area of triangle = $\frac{1}{2} \times 7 \times 6 = 21$

Use the standard formula for the area of a triangle.

30



Draw the rectangle first, marking the length k cm and the width 1 cm. Add the line that creates the square and smaller rectangle (EF in the diagram).

You can see that the square must be 1 cm by 1 cm, so the distance BE must be $(k-1)$ cm.

$$\frac{AD}{AB}=\frac{EF}{BE}$$

Now use the fact that the rectangles are similar.

So

$$\frac{k}{1}=\frac{1}{k-1}$$

Multiply both sides by $(k-1)$, then expand and solve.

$$k(k-1)=1$$

$$k^2-k=1$$

$$k^2-k-1=0$$

$$k=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2}$$

$$k=\frac{1\pm\sqrt{5}}{2}$$

But

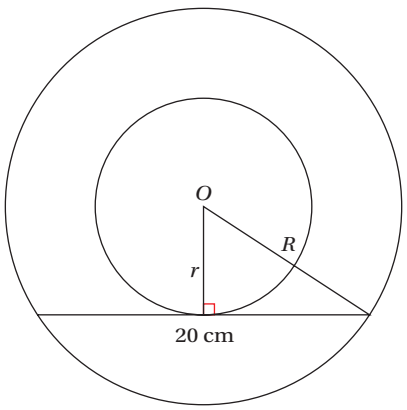
$\frac{1-\sqrt{5}}{2}<0$ and k is a length, so cannot be negative.

So

$$k=\frac{1+\sqrt{5}}{2}$$

For interest, this number is called the *Golden Ratio* or *Golden Section*.

31



We need to think about the radius of each circle if we are going to find an area, but we don't need to find the actual value of either radius.

$r^2 + 10^2 = R^2$ [1]

The triangle is right-angled because a radius and a tangent will always meet at a right angle. Use Pythagoras.

Area of ring $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

What did the original question ask you for? Use this and the diagram as a guide.

But equation [1] tells us that
$R^2 - r^2 = 10^2 = 100$

It is always worth reminding yourself what the original question was asking you to do.

So area of ring $= \pi(100)$

$= 100\pi$ cm^2